KENDRIYA VIDYALAYA SANGATHAN
REGIONAL OFFICE ERNAKULAM

STUDY MATERIAL

CLASS XII

MATHEMATICS

Term : I

Based on latest CBSE Exam Pattern for the Session 2021-22

OUR PATRON

HONOURABLE DEPUTY COMMISSIONER

KVS RO ERNAKULAM REGION

SHRI R. SENTHIL KUMAR

GUIDING FORCE

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Assistant Commissioner
KVS RO Ernakulam

Shri Santhosh Kumar N
Assistant Commissioner
KVS RO Ernakulam

Coordinated By

Smt. BINDU LEKSHMY P L
PRINCIPAL
KV CHENNEERKARA

OUR TEAM OF DEDICATED TEACHERS WHO CONTRIBUTED.

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1.	Mr. Mohan Kumar	KV2, Kochi.
2.	Mrs. Preeta. M.	KV2, Kochi.
3.	Mrs. Jyolsna.	KV1, Calicut.
4.	Mrs. Mini.S.	KV, Kalpetta.
5.	Mrs.Beena Prince	KV Port Trust
6.	Mrs. Shiji Natarajan.	KV Thrissur.
7.	Mrs. Beena Chacko.	KV Ernakulam.
8.	Mrs. Bindu. T.K.	KV Adoor.
9.	Mrs. Beena Joseph	KV CRPF, Pallipuram.
10	Mrs. Sreelatha. V.	KV Pangode.
11.	Mrs. Kunjamma Pius.	KV2, Calicut.
12.	Mr Sreejith. S.	KV1, Calicut
13.	Mrs. Rajalakshmy.	KV No.1, Palakkad.
14.	Mrs. Jayasree	KV,INS, Dronacharya.
15.	Mr. Harikumar.	KV Pattom Shift-I.
16.	Mrs. Aleyamma Idicula.	KV Ottappalam.
17.	Mr. Krishnadas.	KV Kanjikode.
18	Mrs. Sunitha.S.	KV Pangode.
19.	Mrs. Mini Antony.	KV No.1, Kochi.
20	Mrs. Sreedevi.	KV NTPC, Kayamkulam.
21.	Mrs. Sophyamma Verghese.	KV1, Calicut
22.	Mrs. Mercy Mathew.	KV Port Trust Kochi.
23	Mr. Shanu Kumar.	KV Thrissur.
24.	Mrs. Letha .K. Nair.	KV, Akkulam.
25.	Mrs. Sreelekha.	KV Ernakulam
26.	Mr. Santosh B.	KV RB, Kottayam.
27	Mrs. Darley John.	KV Adoor Shift- I
28	Mr. Sreekumar.	KV Adoor Shift – II.
30	Mrs. Preema Paul.	KV, Kannur.
31	Mrs. Jaya David.	KV, Ottappalam
32	Mr. Shiva Kumar	KV, Pattom Shift- II.
33	Mrs. Gracy Franklin.	KV No.1, Kochi.
34	Mr. Sreekumar.U.	KV, Chenneerkara.

आर सेन्थिल कुमार
उपायुक्त

R. Senthil Kumar
Deputy Commissioner

केन्द्रीय विद्यालय संगठन,
क्षेत्रीय कार्यालय, एरणाकुलम

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F.31/Acad/KVS(EKM) *Dated: 01.11.2021*

<u>Message</u>

I feel immense pleasure to publish the study material for class XII Maths. This support material is prepared incorporating all the recent changes in curriculum and assessment process made by CBSE. I am sure it will definitely be of great help to class XII students of all Kendriya Vidyalayas.

Getting acquainted with the latest changes will help students to prepare well for the board examination and enable students to face case based and Multiple-Choice Questions with confidence. This support material has been prepared by a team of dedicated and veteran teachers with expertise in their respective subjects.

The Support material contains all the important aspects required by the students- the design of question paper, term wise split up syllabus, summary of all the chapters, important formulas, Sample question papers, problem solving and Case study questions.

I hope that this Support Material will be used by students and teachers as well and will prove to be a good tool for quick revision.

I would like to express my sincere gratitude to the In- charge principal and all the teachers who have relentlessly worked for the preparation of this study material. Their enormous contribution in making this project successful is praiseworthy.

Meticulous planning blended with hard work, effective time management and sincerity will help the students to reach the pinnacle of success.

Wish you all the best

(R Senthil Kumar)

Mrs. Bindu Lekshmy PL
Principal
Kendriya Vidyalaya
Cheneerkara

A Foreword to Students ....

As per the new CBSE pattern for class 12, the board exams will be held twice in the same year with a reduced syllabus and two-term exams; first, **MCQ-based** and the second subjective based. KVSRO Ernakulam Region is presenting this material to equip students to get accustomed to the New Pattern of Assessment and Evaluation.

This Study material is provided with a view to make your attention focussed on the Term I Examination for Class XII Mathematics. Here are a few Tips

1. Know your Syllabus and Devise a Schedular. ...
2. Know the pattern and practice each type of question. ...
3. Kickstart with NCERT syllabus. ...
4. Enhance your analytical knowledge.
5. Regular and Continuous Practice.

   In this Study material you are provided with all first hand information which you need to know about Syllabus ,Pattern of Question Paper and Scheme of Evaluation. Chapter-wise Learning Materials . CBSE Sample Paper and Two Model Papers with Answer keys are also included as your Practice Materials.

CBSE SENIOR SCHOOL CERTIFICATE EXAMINATION

SUBJECT: MATHEMATICS (Code: 041)

DAY, DATE & TIME OF EXAMINATION

MONDAY, 6TH DECEMBER

11.30 AM – 01.00 PM.

EXAMINATION TIPS:

1. On the day of Examination, report to the Examination Centre well in advance. 2. Since every question carries equal weightage of Marks, more emphasis should not be given to any one particular question during the Examination.

3. Manage your time. Budget your time to answer each question, review your answers, and transfer them to your answer sheet.

4. Since there is no negative marking, answer all questions as per the choices given in the question paper. Even if you don’t know an answer, make an educated guess. There is a chance you might get the marks. If you don’t try, you are guaranteed to get zero. So find time to answer all required number of Questions.

5. Be relaxed and remain focussed during the Examination. This will naturally optimise your performance.

HAPPY LEARNING

INDEX OF CONTENTS.

REDUCED AND BIFURCATED SYLLABUS

ACADEMIC YEAR 2021–22

MATHEMATICS CLASS-XII (2021-22)

TERM – I

One Paper.

Time : 90 minutes Max.Marks : 40.

SL.NO.	UNITS	MARKS
I	Relations and Functions.	08
II	Algebra.	10
III	Calculus.	17
IV	Linear Programming.	05
	Total (Theory)	40
	Internal Assessment	10
	Total.	50.

Unit-I: Relations and Functions-

1. Relations and Functions.

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions

2. Inverse Trigonometric Functions.

Definition, range, domain, principal value branch.

Unit-II: Algebra-

1. Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non- commutativity of multiplication of matrices, Invertible matrices; (Here all matrices will have real entries).

2. Determinants

Determinant of a square matrix (up to 3×3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus -

1. Continuity and Differentiability

Continuity and differentiability, derivative of composite functions, chain rule, derivative of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions. Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives :

Applications of derivatives: increasing/decreasing functions, tangents and normals, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

Unit-IV: Linear Programming-

1. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems. Graphical method of solution for problems in two variables, feasible and infeasible regions (bounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Internal Assessment	**10 Marks**
Periodic Test	5 Marks
Mathematics Activities: Activity file record +Term end assessment of one activity & Viva	5 Marks.

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UNIT : I - RELATIONS AND FUNCTIONS.

1. Relations and Functions.
2. Inverse Trigonometric Functions

UNIT – I

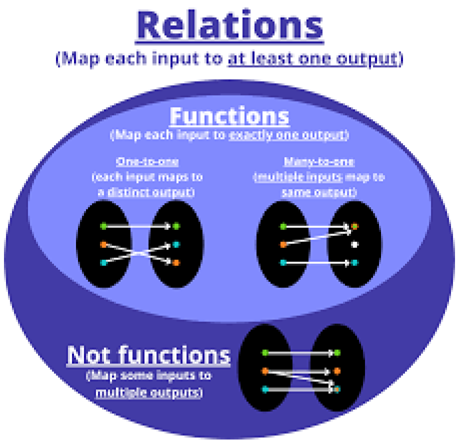
1. RELATIONS AND FUNCTIONS.

1. GIST OF THE LESSON.

2. MIND MAPPING- FOR QUICK REVISION.

3. MULTIPLE CHOICE QUESTIONS WITH ANSWER KEY.

4.CASE STUDY AND ASSERTION & REASONING : MCQs WITH ANSWER KEY.



PART : 1

RELATIONS AND FUNCTIONS.

GIST OF THE LESSON

1.Relation: If A and B are two non-empty sets, then any subset R of A X B is a relation from A to B

If (a ,b) ∈ R, we write aRb .

2. Domain of R is the set of all first coordinates of elements of R, **Range** of R is the set of all second coordinates of R

3.Types of relations

(a)**Empty relation**: A relation R in a set A is called empty relation if no element of A is related to any element of A i.e. R= ϕ ⊂ A X A

(b) **Universal relation**:A relation R in a set A is called universal relation if each element of A is related to every element of A i e. R= A X A

(c) **Reflexive relation**: A relation R in a set A is called reflexive , if (a , a) ∈ R , for every a ∈A

(d) **Symmetric relation**: A relation R in a set A is called symmetric, if (a , b) ∈ R ⇒(b, a) ∈ R for every a, b ∈ A

(e) **Transitive relation** : A relation R in a set A is called transitive ,if (a, b) ∈ R and (b, c) ∈ R ⇒ (a, c) ∈ R , for every a, b, c ∈ R

(f) **Equivalence relation**: A relation R in a set A is called an equivalence relation if R is reflexive , symmetric and transitive

4.Equivalence class: Let R be an equivalence relation on a non-empty set A. For all a ∈ A , the equivalence class of ' a ' is defined as the set of all such elements of A which are related to 'a ' under R . It is denoted by [a]

[a] = { x ∈ A : (x, a) ∈ R }

5.Function : A relation f from a set A to a set B is said to be a function if every element of A has one and only one image in set B. Set A is called the domain of the function f and B is known as co-domain of f . The set of images is called range of f.

6. One-one function (or injective): A function f:A →B is said to be one-one, if the images of distinct elements A under f are distinct, i. e. , for every a, b ∈ A , f(a) = f(b) implies a=b. Otherwise f is called many-one.

7. Onto function (or Surjective) :A function f:A→ B is called onto , if every element of B is the image of some element of A under f, i.e., for every b ∈ B , there exists an element a in A such that f(a) = b.

f:A → B is onto if and only if Range of f = B (co-domain)

8. Bijective function : A function f is bijective,if f is both one-one and onto.

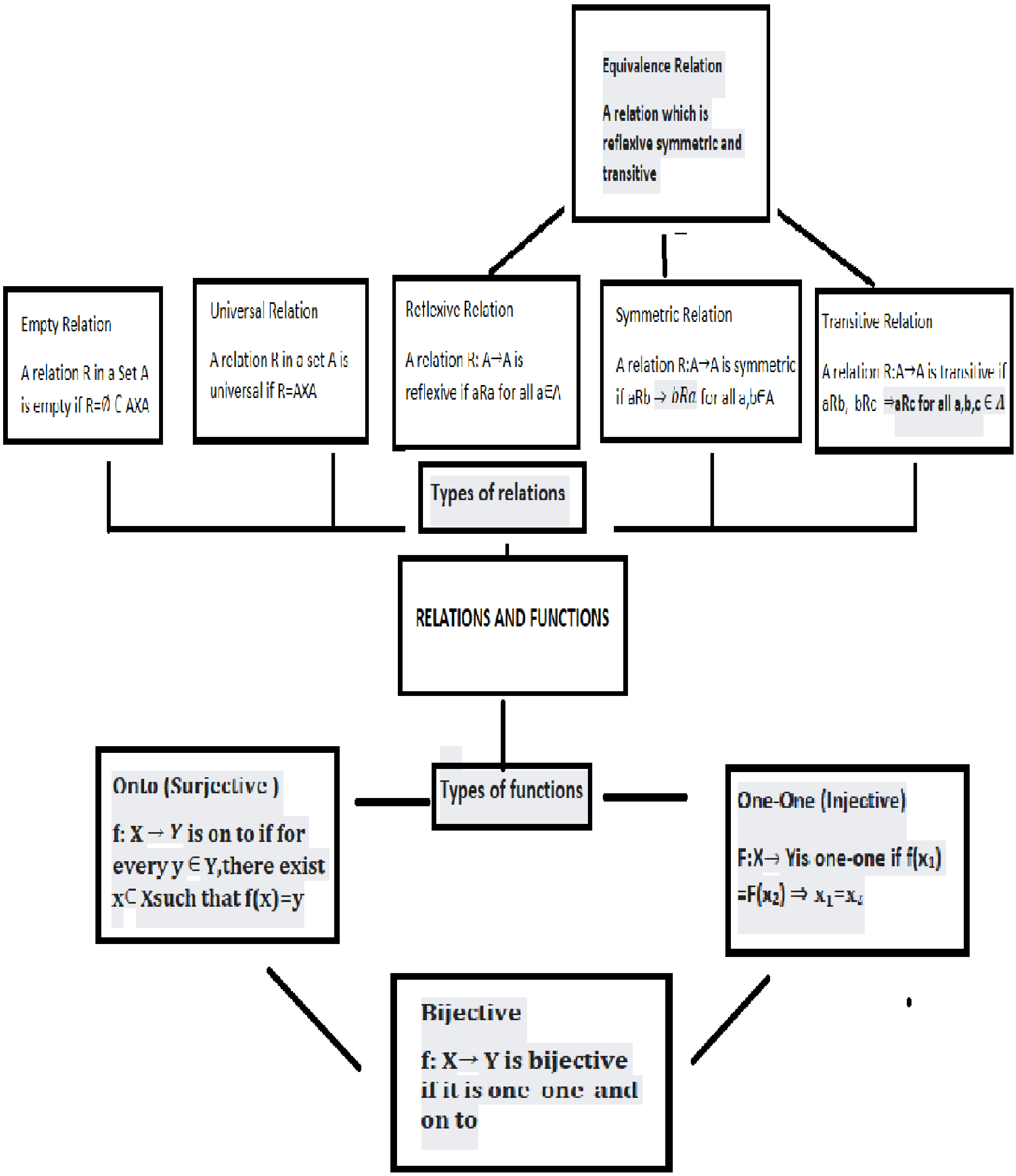
9..Let A and B be two non-empty finite sets such that n(A)= m and n(B) = n .Then the number of **functions** from A to B is $\boldsymbol{n^m}$

10. Let A and B be two non-empty finite sets such that n(A)= m and n(B) = n. Then the number of **one-one functions** from A to B = $\begin{Bmatrix} \mathbf{0} & \boldsymbol{if\ m > n} \\ \mathbf{nPm} & \boldsymbol{if\ m \leq n} \end{Bmatrix}$ where **nP_m =** $\frac{\boldsymbol{n}!}{(\boldsymbol{n-m})!}$

11.Let A be any finite set having n elements. Then the number **on-to functions** from A to A is **n**!

12. Let A and B be two non-empty finite sets such that n(A)= m and n(.B) = n. Then then number of Bijective functions from A to B is = $\begin{Bmatrix} \mathbf{n}! & \boldsymbol{if\ m = n} \\ \mathbf{0} & \boldsymbol{if\ m \neq n} \end{Bmatrix}$

MIND MAPPING -RELATIONS AND FUNCTIONS



RELATIONS AND FUNCTIONS- MULTIPLE CHOICE QUESTIONS

1	The Relation R defined on the set A = {1,2,3}by R = {(1,1), (3,1), (2,2)(1,3),(2,1)(3,3)}. Which ordered pair must be included to make it an equivalence relation a) (2,3) b) (3,2) c) (1,2) d) None of these
2	Let A = {5, 6, 7 , 8} Let R be the equivalence relation on A xA defined as (a, b)R (c, d) iff a + d = b + c. Then the equivalence class of (5,7) is a) {(5,7)} b){(5,5),(6,8)} c){(5,6),(7,8)} d){(5,8),(6,7)}
3	Consider a Relation R defined on the set {x$\in Z: 0 \leq x \leq 12\}$,given by R={(a,b):$\|a-b\|$is a multiple of 3}. Find the set of all relations related to 2 a) {2,5,8,11} b) { 5, 8,11} c) { 1, 3, 5} d) {2, 8, 10}
4	Consider a Relation R defined on Z given by R={(a,b):$(a-b)$is even,a , b$\in Z$ }. is a) Reflexive but not symmetric b) Neither symmetric nor transitive c) Symmetric but not transitive d) equivalence relation
5	The relation R in the set { a, b, c} given by R= { (a,a),(b, b),(c,c),(a,b),(b,c) } a) Not Reflexive but symmetric b) Neither symmetric nor transitive c) Symmetric but not transitive d) equivalence relation
6	Let L be the set of all lines in a plane and R be the relation in L defined as R= {(l,m): l is perpendicular to m} then R is a) Equivalence relation b) symmetric and transitive

	c) symmetric but neither reflexive nor transitive d) Reflexive
7	Let R be the relation in the set N given by R= { (a,b): b-a = 2, b>6} choose the correct answer (2, 4)$\in R$ b) (3, 8) $\in R$ c) (6,8)$\in R$ d) (8, 7)$\in R$
8	Let A={ a,b,c } ,Then the number of relations containing (a,b) and (a,c) which are reflexive and symmetric but not transitive is a)1 b) 2 c) 3 c) 4
9	Let A={ 1,2,3 } ,Then the number of equivalence relations containing (3, 2) is a) 1 b) 2 c) 3 c) 4
10	A relation R in the set A= { 1,2,3,4} is defined as R={(1,1),(1,2),(2,2),(2,1),(2,3),(3,3)(2,4),(4,4)(3,2),(1,3),(3,1)} Which element shall be removed from the Relation a)((4,4) b)(3,2) c) (2, 4) d) (3,1)
11	The number of all one one functions from set A ={ 1,2,3,4} to itself is: a) 3 b) 6 c) 24 d) 16
12	If the set A has 3 elements and set B has 4 elements then the number of one one and onto mappings from A to B is a) 0 b) 12 c)12! d) 3!
13	Which of the following functions from Z to Z is a bijection a) f(x)= $x^2 + 1$ b) f(x)= x + 2 c) f(x) = x^3 d)2x+1
14	Let f: R →R defined as f(x)=x^3choose the correct answer

	a) f is one one onto b) f is many one onto c) f is one one not onto d) f is neither one one nor onto
15	Let f: R $\rightarrow$ R given by f(x) = $\|x\|$ choose the correct answer a) f is injective and surjective b) f is many one surjective c) f is injective but not surjective d) f is neither injective nor surjective
16	Let A and B be sets f: AxB→BxA f(a,b)= (b,a). Choose the correct answer a) f is one one onto b) f is many one onto c) f is one one not onto d) f is not one one but onto
17	Find the number of one one functions from the set of A={1,2,3} to itself a) 3 b) 9 c) 6 d) 2
18	Let A= {1,2,3} Then number of relations containing (1, 2),(2, 3) which are reflexive and transitive but not symmetric is? a)3 b) 4 c) 1 d) 5
19	Find how many number of equivalence relations in the set {a,b,c} containing (a, b) and (b, a) a) 3 b) 1 c) 4 d) 2
20	Let $f: R \rightarrow R \quad defined\ by\ f(x) = x^4$. Choose the correct answer a) f is neither one-one nor onto b) f is one one but not onto c) f is many one onto d) None of these

Answers

1	c) (1, 2)	11	c) 24
2	c) (5, 6), (7, 8)	12	a) 0
3	a) {2,5,8,11}	13	b) x+2
4	d) Equivalence	14	a) f is one one onto
5	b) Neither symmetric nor transitive	15	d) f is neither injective nor surjective
6	c) Symmetric but not refiexive and transitive	16	a) f is one one onto
7	c) (6, 8)	17	c) 6
8	a) 1	18	b) 4
9	b) 2	19	d) 2
10	c) (2,4)	20	a) f is neither one-one nor onto

CASE STUDY QUESTIONS : RELATIONS AND FUNCTIONS

I An organization conducted car race under two different categories boys and girls. Total there are 300 participants. Among all of them 2 from category 1 and 3 from category 2 were selected for the final race. Raju forms two sets B and G for these participants B = {b1,b2} G = {g1 , g2, g3} where B represents the set of boys and G set of girls ,who were selected for the final race.

Based on the above information answer the following questions

1.Raju wishes to form all the relationship possible from B to G. How many such relations are possible.

(a) 2^6 (b) 2^5 (c) 2^3 (d) 0

2. Let $R: G \to G$ defined by { (x,y) : x and y are students of same sex} then the relation R is

(a) equivalence (b) reflexive only (c) reflexive and symmetric but not transitive (d) reflexive and transitive but not symmetric.

3. Raju wants to know among those relations how many functions can be formed from B to G

(a) 2^2 (b)2^4 (c)2^3 (d) 3^2

4.Let $R: B \to G$ defined by R = { (b1,g1) , (b2,g2)} then R is

(a) injective (b) surjective (c) neither injective nor surjective (d) bijective

5. Raju wants to find the number of surjective function from B to G. How many such functions can be possible.

(a) 0 (b) 2! (c) 3! (d) 0!

II. Arjun visited the exhibition along with their family the exhibition had a huge swing. Arjun found that the swing traced the path of a parabola $y = x^2$

1.Let f: $R \rightarrow R$ defined by $f(x) = x^2$ is

(a) Neither surjective nor injective (b) Surjective (c) injective (d) bijective

2.f:$N \rightarrow N$ defined by $f(x) = x^2$ is

(a) Neither surjective nor injective (b) Surjective (c) injective (d) bijective

3.Let f:$\{1,2,3 \ldots\}$ → {1,4,9…} defined by $f(x) = x^2$

(a) bijective (b) surjective but not injective (c) injective but not surjective (d) neither surjective nor injective

4.f:$\{-3,-2,-1,0,1,2,3\} \rightarrow R$ defined by $f(x) = x^2$ the range of the function is

(a) {1,4,9} (b) {0,1,4,9} (c) {-9,-4,-1,0,1,4,9} (d) $\{-3,-2,-1,0,1,2,3\}$

5. If one of his family member is interested in the giant wheel. The path of giant wheel is $x^2 + y^2 = 100$

If R:$\{1,2,3,4,5,6,7,8\} \rightarrow R$ the find the range of R.

(a) $\{1,2,3,4,5,6,7,8\}$ (b) {6,8} (c) {1,4,9,16,25,36,49,64}
(b) (d) $\{\sqrt{99}$, $\sqrt{96}, \sqrt{91}\sqrt{84}\sqrt{75}, \sqrt{64}, \sqrt{51}$, $\sqrt{36}\}$

ANSWERS

I

1)a 2)a. 3) d 4) a 5) a

II

1)a 2)c 3)a 4) b 5) d

ASSERTION AND REASONING QUESTIONS

The following questions consist of two statements one labeled as Assertion (A) and the other labeled as Reason (R) .You are to examine these two statement carefully and design if assertion A and reason R are individually true and if so whether the reason R is the correct explanation for the given assertion Select answer from the following option

(a) A is true and R is false
(b) A is false but R is true
(c) Both A and R are true and R is the correct explanation of A
(d) Both A and R are true but R is not the correct explanation of A

1.Consider the set {1,3,5}

Assertion (A) : The number of reflexive relations on set A is 2^9

Reason (R): A relation is said to be reflexive if xRx,$\forall x \epsilon A$

2.A function f:$Z \rightarrow Z$ defined as f(X) = X^3

Assertion(A) :f: $Z \rightarrow Z$is injective

Reason(R) :f:$A \rightarrow B$ is injective if every element of A has different image in B

ANSWERS

1.b 2.c ==

UNIT: I- RELATIONS AND FUNCTIONS

PART: 2. INVERSE TRIGNOMETRIC FUNCTIONS.

1. Gist of the Lesson.

2. Mind Mapping for Quick Revision.

3. Multiple Choice Questions.

4. Case Study Questions.

5. Assertion and Reasoning Type.

PART 2

INVERSE TRIGONOMETRIC FUCTIONS

GIST OF THE LESSON

- Domain and range of trigonometric functions
- Graph of inverse trigonometric functions with principal value branch
- Domain and range of inverse trigonometric functions
- Properties of inverse trigonometric functions
- Reducing inverse trigonometric functions in simplest form
- Use of properties of inverse trigonometric functions in different questions

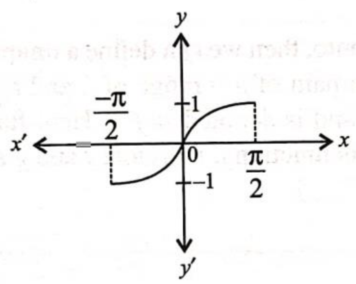
Basic concepts

Table for domain and range of inverse trigonometric functions

Functions	Domain	Range
(i) sine	R	$[-1, 1]$
(ii) cosine	R	$[-1, 1]$
(iii) tangent	$R - \{x : x = (2n+1)\frac{\pi}{2}, n \in Z\}$	R
(iv) cosecant	$R - \{x : x = n\pi, n \in Z\}$	$R - [-1, 1]$
(v) secant	$R - \{x : x = (2n+1)\frac{\pi}{2}, n \in Z\}$	$R - [-1, 1]$
(vi) cotangent	$R - \{x : x = n\pi, n \in Z\}$	

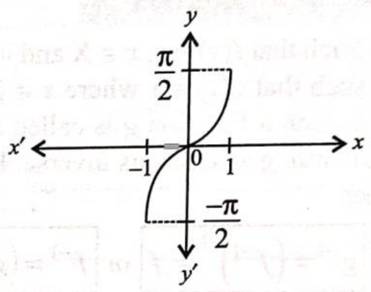
Graph of inverse trigonometric functions with principal value branch

Graph of $y = \sin x$ and $y = \sin^{-1} x$



$y = \sin x : \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \rightarrow [-1,1]$

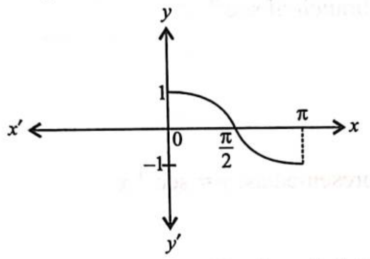
Graph of $y = \sin x$



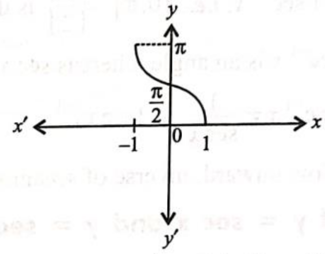
$y = \sin^{-1} x : [-1,1] \rightarrow \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Graph of $y = \sin^{-1} x$

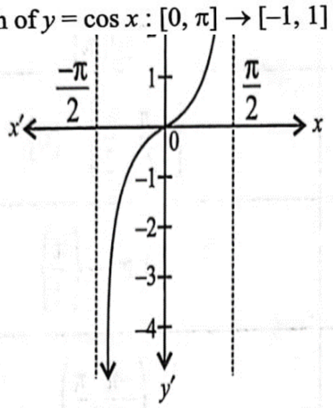
Graph of y = cos x and y = $\cos^{-1}$ x



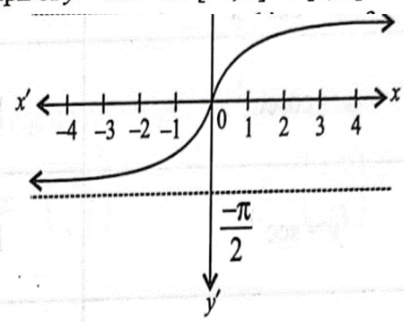
Graph of $y = \cos x : [0, \pi] \to [-1, 1]$



Graph of $y = \cos^{-1} x : [-1, 1] \to [0, \pi]$

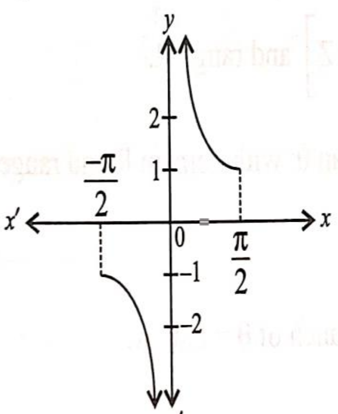


Graph of $y = \tan x : \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \to R$



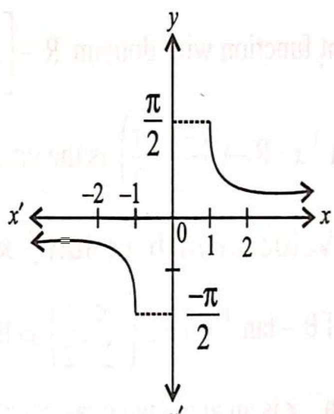
Graph of $y = \tan^{-1} x : R \to \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Graph of y = cosec x and y = cosec^{-1} x



Graph of

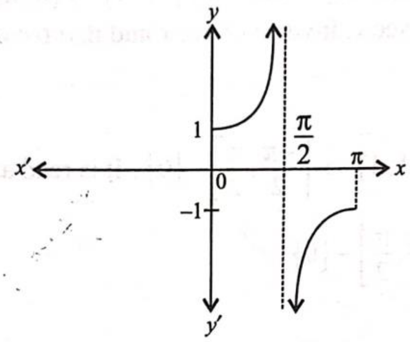
$y = \text{cosec } x = \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\} \to (-1,1)$



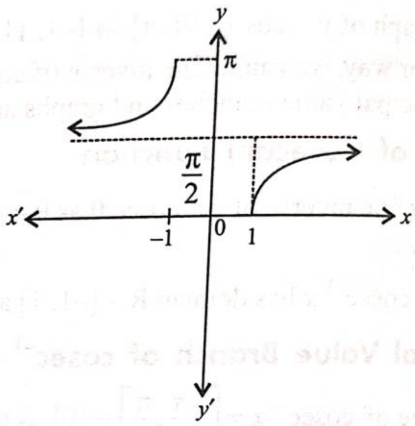
Graph of

$y = \text{cosec}^{-1} x = R - (-1,1) \to \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Graph of y = sec x and y = sec⁻¹ x

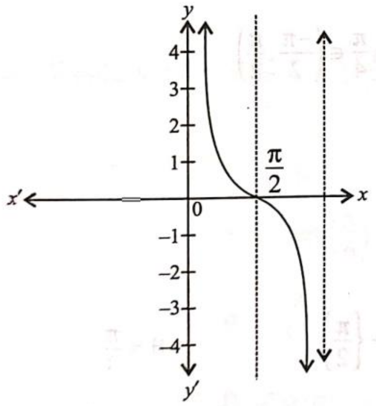


Graph of $y=\sec x:[0,\pi]-\left\{\frac{\pi}{2}\right\}\to \mathrm{R}-(-1,1)$

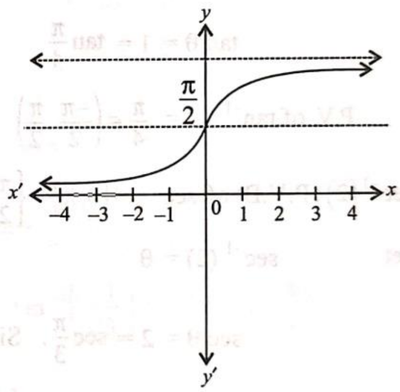


Graph of $y=\sec^{-1}x=\mathrm{R}-(-1,1)\to[0,\pi]-\left\{\frac{\pi}{2}\right\}$

Graph of y = cot x and y = cot⁻¹ x



Graph of $y=\cot x:\mathrm{R}-\{x:x=n\pi,\ n\in \mathrm{Z}\}\to \mathrm{R}$



Graph of $y=\cot^{-1}x:\mathrm{R}\to(0,\pi)$

The Domain and Ranges (Principal Value Branches) of Inverse Trigonometric Functions

Inverse Trigonometric Functions	Domains	Ranges (Principal Value Branches)
$y=\sin^{-1}x$	$[-1, 1]$	$\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$
$y=\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y=\operatorname{cosec}^{-1}x$	$\mathrm{R}-(-1, 1)$	$\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$
$y=\sec^{-1}x$	$\mathrm{R}-(-1, 1)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$
$y=\tan^{-1}x$	R	$\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$
$y=\cot^{-1}x$	R	$(0, \pi)$

Properties of Inverse Trigonometric Functions

1. **Properties of Inverse Trigonometric Functions**

(i)
- $\sin^{-1}(\sin x) = x$; • $\sin(\sin^{-1}x) = x$
- $\cos^{-1}(\cos x) = x$; • $\cos(\cos^{-1}x) = x$
- $\tan^{-1}(\tan x) = x$; • $\tan(\tan^{-1}x) = x$
- $\text{cosec}^{-1}(\text{cosec}\,x) = x$; • $\text{cosec}(\text{cosec}^{-1}x) = x$
- $\sec^{-1}(\sec x) = x$; • $\sec(\sec^{-1}x) = x$
- $\cot^{-1}(\cot x) = x$; • $\cot(\cot^{-1}x) = x$

(ii)
- $\sin^{-1}\left(\frac{1}{x}\right) = \text{cosec}^{-1}x$; • $\text{cosec}^{-1}\left(\frac{1}{x}\right) = \sin^{-1}x$
- $\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}x,\ x > 0$; • $\cot^{-1}\left(\frac{1}{x}\right) = \tan^{-1}x$
- $\sec^{-1}\left(\frac{1}{x}\right) = \cos^{-1}x$; • $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x$

(iii)
- $\sin^{-1}(-x) = -\sin^{-1}x$; • $\cos^{-1}(-x) = \pi - \cos^{-1}x$
- $\tan^{-1}(-x) = -\tan^{-1}x$; • $\sec^{-1}(-x) = \pi - \sec^{-1}x$
- $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$; • $\cot^{-1}(-x) = \pi - \cot^{-1}x$

(iv)
- $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$; • $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}$
- $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

(v)
- $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$, if $xy < 1$
- $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$, $xy > -1$

(vi) $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x.$

2. (i) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

(ii) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$

(iii) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

(iv) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$

Inverse Trigonometric Functions in Simplified Form

Some Standard Substitutions

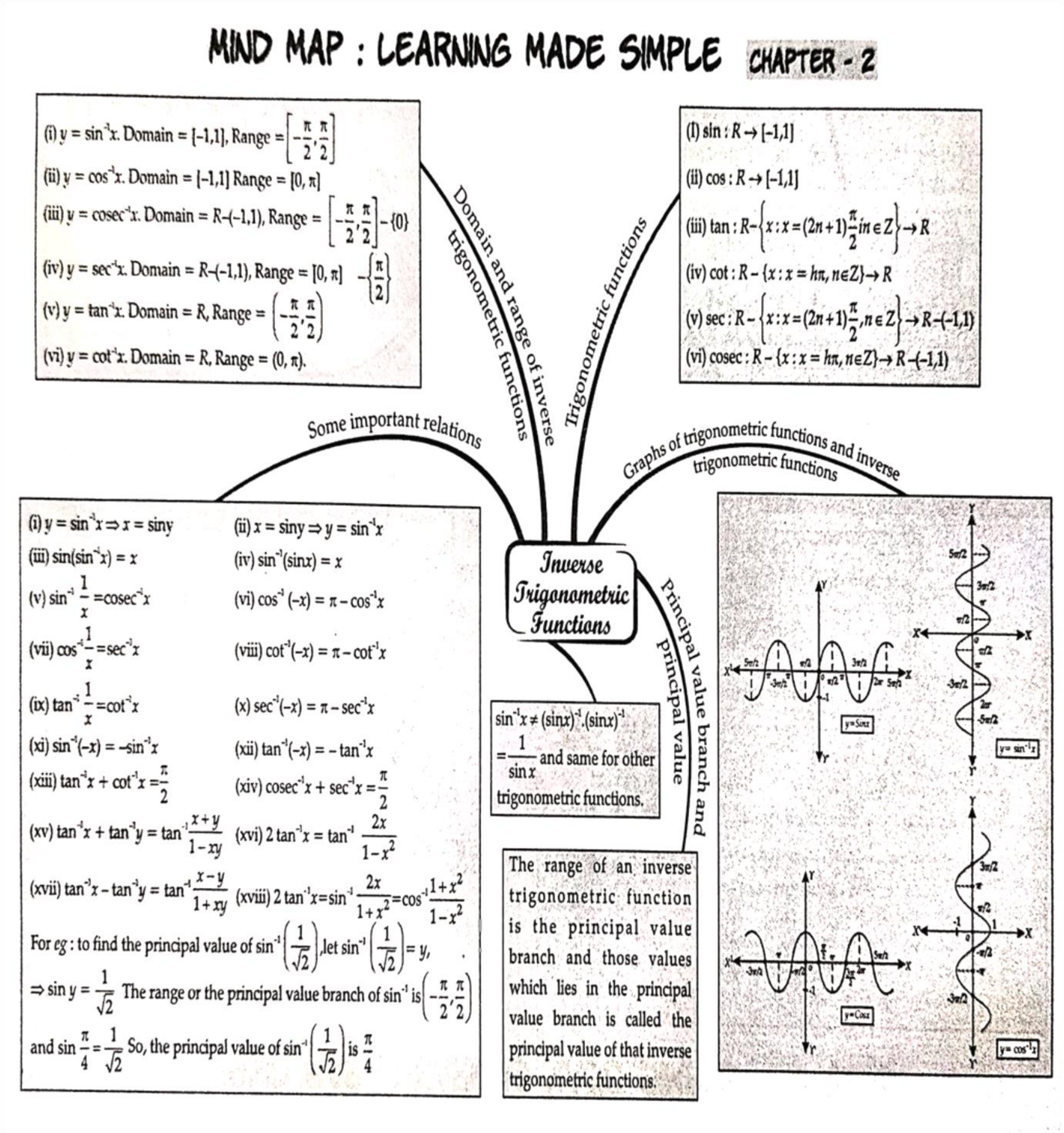
(i) For $\sqrt{a^2 - x^2}$; we substitute $x = a\sin\theta$ or $x = a\cos\theta$.

(ii) For $\sqrt{a^2 + x^2}$; we substitute $x = a\tan\theta$ or $x = a\cot\theta$.

(iii) For $\sqrt{x^2 - a^2}$; we substitute $x = a\sec\theta$ or $x = a\,\text{cosec}\,\theta$.

(iv) For $\sqrt{a+x}$ and $\sqrt{a-x}$ occurring together or separately, we substitute $x = a\cos\theta$ or $a\cos 2\theta$.

INVERSE TRIGONOMETRIC FUNCTIONS:MIND MAPPING.



MULTIPLE CHOICE QUESTIONS

1. The principal value of $\cos^{-1}(\cos\frac{17\pi}{5})$ is

   a) $\frac{17\pi}{5}$ b) $\frac{2\pi}{5}$ c) $\frac{3\pi}{5}$ d) $\frac{-2\pi}{5}$

2. The value of $\tan^{-1}(1)+\cos^{-1}(\frac{-1}{2})$ is

   a) $\frac{3\pi}{4}$ b) $\frac{11\pi}{12}$ c) $\frac{2\pi}{3}$ d) $\frac{13\pi}{12}$

3. Using the principal value, find the value of $\tan(\sec^{-1}(\frac{-2}{\sqrt{3}}))$.

   a) $-\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) $\sqrt{3}$ d) $\frac{-1}{\sqrt{3}}$

4. $sin\left(\frac{\pi}{3} - sin^{-1}\left(\frac{-1}{2}\right)\right)$ is equal to

   a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{-1}{2}$ d) 1

5. The value of $tan^{-1}(\sqrt{3}) - cot^{-1}(-\sqrt{3})$

   a) π b) $\frac{\pi}{2}$ c) 0 d) $2\sqrt{3}$

6. If $\sec^{-1}\left(\frac{a}{5}\right)+\sin^{-1}\left(\frac{5}{b}\right) = \frac{\pi}{2}$, then

   a) ab=1 b) a=b c) $a=b^2$ d) none of these

7. If $\tan^{-1}\left(\frac{a}{x}\right)+\tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$, then the value of x is

   a) ab b) 1 c) $\sqrt{ab}$ d) none of these

8. The Value of $\sin(\tan^{-1}\frac{3}{4}) + cot\left(tan^{-1}\frac{5}{12}\right)$

   a) $\frac{7}{6}$ b) 3 c) $\frac{5}{6}$ d) 5

9. Evaluate: $\sin^{-1}(\cos(\sin^{-1}\frac{\sqrt{3}}{2})$

   a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

10. Find the value of $\sec(\tan^{-1}(\frac{y}{2}))$

    a) $\frac{4+y^2}{2}$ b) $\frac{4-y^2}{2}$ c) $\sqrt{\frac{4-y^2}{2}}$ d) $\sqrt{\frac{4+y^2}{2}}$

11. The simplest form of $\cos^{-1}\left(\frac{12}{13}\cos x - \frac{5}{13}\sin x\right)$ is

a) $\sin^{-1}\left(\frac{12}{13}\right) + x$ b) $\cos^{-1}\left(\frac{5}{13}\right) + x$

c) $\cos^{-1}\left(\frac{12}{13}\right) + x$ d) none of this

12. The value of expression $\tan\left(\frac{\sin^{-1}x+\cos^{-1}x}{2}\right)$

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) 0 d) 1

13. The principal value of the expression $\cos^{-1}[\cos(-680°)]$ is

a) $\frac{\pi}{9}$ b) $\frac{2\pi}{9}$ c) $-\frac{\pi}{9}$ d) $\frac{34\pi}{9}$

14. The domain of $y = \sin^{-1}(x^2-8)$ is

a) $[-3, -\sqrt{7}] \cup [\sqrt{7}, 3]$ b) $[-3, -\sqrt{7}] \cap [\sqrt{7}, 3]$

c) $(-\infty, \infty)$ d) none of these

15. The domain of function $y = \cos^{-1}(2x-1)$ is

a) $(0, \infty)$ b) $[0, \infty)$ c) $(-1,1)$ d) none of these

16. If $4\tan^{-1}x+\cot^{-1}x=\pi$, then x is

a) 1 b) $\frac{1}{\sqrt{3}}$ c) 0 d) $\sqrt{3}$

17. The value of $\tan^2(\sec^{-1}2) + \cot^2(\text{cosec}^{-1}3)$ is

a) 5 b) 13 c) 11 d) 10

18. The simplified form of $\cos(\tan^{-1}(\sin(\cot^{-1}x)))$ is

a) $\frac{x}{\sqrt{1+x^2}}$ b) $\frac{\sqrt{2+x^2}}{\sqrt{1+x^2}}$

c) $\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$ d) none of these

19. Using principal value evaluate $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

a) 0 b) $\frac{4\pi}{3}$ c) π d) $\frac{\pi}{2}$

20. If $\cos^{-1}x = \frac{5}{7}$, then $\tan^{-1}x$ is

a) $\frac{5}{\sqrt{24}}$ b) $\frac{\sqrt{24}}{5}$ c) $\frac{7}{\sqrt{24}}$ d) $\frac{\sqrt{24}}{7}$.

CASE STUDY QUESTIONS

1. A group of students of class XII visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Raj path (formerly called the Kingsway), is about 138 feet (42 m) in height.

Read the paragraph and answer the following questions:

a) What is the angle of elevation if they are standing at a distance of 42m away from the monument?

(i) $\tan^{-1}1$

(ii)$\sin^{-1}1$

(iii) $\cos^{-1}1$

(iv)$\sec^{-1}1$

b) They want to see the tower at an angle of $\sec^{-1} 2$. At what distance should they stand from the monument?

(i) 42m

(ii) 20.12m

(iii)25.24m

(iv)24.24m

c) If the altitude of the Sun is at $\cos^{-1}$ (1/2) then the height of the vertical tower that will cast shadow of length 20m is

(i) $20\sqrt{3}$m

(ii) $20/\sqrt{3}$ m

(iii) $15\sqrt{3}$ (iv) $15/\sqrt{3}$ m

d) The ratio of the length of an electric pole and its shadow is 1:2.The

angle of elevation of the sun is :

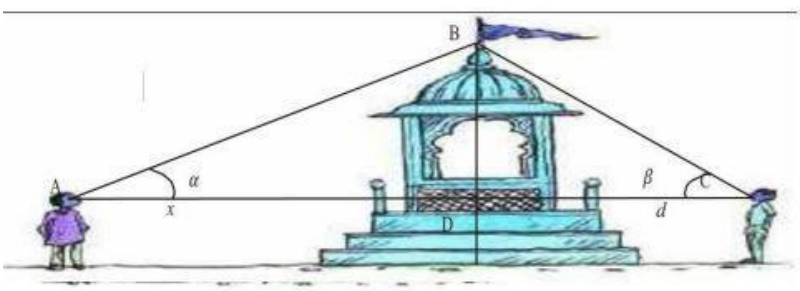
(i) $\sin^{-1}(1/2)$

(ii) $\cos^{-1}(1/2)$

(iii) $\tan^{-1}(1/2)$

(iv) $\cot^{-1}(1/2)$

2. Two men on either side of a temple of 30 meters high observe its top at the angles of elevation $\propto$ and β respectively. (as shown in the figure below). The distance between the two men is 40√3 meters and the distance between the first person A and the temple is30√3 meters. Based on the above information answer the following:



a) $\angle CAB = \propto$

(i) $\sin^{-1}(1/\sqrt{3})$

(ii) $\sin^{-1}(1/2)$

(iii) $\sin^{-1}(2)$

(iv) $\sin^{-1}(\sqrt{3}/2)$

b) $\angle CAB = \propto$ =

(i) $\cos^{-1}(1/5)$

(ii) $\cos^{-1}(2/5)$

(iii) $\cos^{-1}(\sqrt{3}/2)$

(iv) $\cos^{-1}(4/5)$

c) $\angle BCA = \beta$

(i) $\tan^{-1}(1/2)$

(ii) $\tan^{-1}(2)$

(iii) $\tan^{-1}(1/\sqrt{3})$

(iv) $\tan^{-1}(\sqrt{3})$

d) $\angle ABC$ =

(i) $\pi/4$

(ii) $\pi/6$

(iii) $\pi/2$

(iv) $\pi/3$

e) Domain and Range of $\cos^{-1}x$ is

(i) $(-1, 1), (0, \pi)$

(ii) $[-1, 1], (0, \pi)$

(iii) $[-1, 1], [0, \pi]$

(iv) $(-1, 1), [-\pi/2, \pi/2]$

ASSERTION AND REASONING QUESTIONS

Directions .In the following questions ,a statement of Assertion (A) is followed by a statement of Reason (R) .Pick the correct option :

1. **Assertion** (A) : The value of Cos $\left[\frac{\pi}{2} + \sin^{-1}(\frac{-1}{2})\right]$ = $\frac{1}{2}$

Reason (R) : $\sin^{-1}(-\theta)$ = - $\sin^{-1}(\theta)$

a, Both A and R are true and R is the correct explanation of A

b, Both A and R are true but R is not the correct explanation of A

c, A is true, but R is false.

d, A is false but R is true

e, Both A and R are false

2. **Assertion (A)** : Domain of the function $\sin^{-1}(2x-1)$ is $[0,1]$

Reason (R) : Domain of $\sin^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

a, Both A and R are true and R is the correct explanation of A

b, Both A and R are true but R is not the correct explanation of A

c, A is true, but R is false.

d, A is false but R is true

e, Both A and R are false

ANSWER KEY

MCQ

1. c	5. a	9. a	13. b	17.c
2. b	6. b	10. d	14. a	18. c
3. d	7. c	11. c	15. d	19. c
4. d	8. b	12. d	16. b	20. b

CASE STUDY QUESTIONS: Answer key.

1	a)i	b)iv	c)i	d)iii	
2	a)ii	b)iii	c)iv	d)iii	e)iii

ASSERTION AND RESONING QUESTIONS: Answer Key.

Qn.1 Ans. is (b)

$$\left\{\cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \cos\frac{\pi}{3} = \frac{1}{2}\right\}$$

Qn .2 Ans . is (c)

$$\left\{\begin{matrix} D\ of\ \sin^{-1}x\ is\ [-1,1] \Rightarrow -1 \le 2x - 1 \le 1 \Rightarrow 0 \le 2x \le 2 \\ \Rightarrow 0 \le x \le 1 \end{matrix}\right\}$$

UNIT : II -ALGEBRA.

1.MATRICES

2. DETERMINANTS.

PART 3

UNIT: II

1.MATRICES

1. GIST OF THE LESSON.

2. MULTIPLE CHOICE QUESTIONS.

3. CASE STUDY QUESTIONS.

4. ASSERTION & REASONING TYPE

PART : 3

MATRICES- GIST OF THE LESSON.

BASIC CONCEPTS

MATRIX DEFINITION-A set of mn numbers(real or complex)arranged in the form of rectangular array of **m** numbers and **n** columns is called an **mxn** matrix and is denoted by $[a_{ij}]$**mxn**

ROW MATRIX-A Matrix having only one row is called a row matrix

COLUMN MATRIX-A Matrix having only one column is called column matrix

SQUARE MATRIX-A Matrix in which the number of rows are equal to the number of columns, is said a square matrix. Thus an m × n matrix is said to be a square matrix if m=n and is a square matrix of order n

DIAGONAL MATRIX -A square matrix $B = [b_{ij}]$ m × m is said to be a diagonal matrix if all its non diagonal elements are zero, that is a matrix $B = [b_{ij}]$ m × m is said to be a diagonal lmatrix if $b_{ij} = 0$, **when $i \neq j$.**

SCALAR MATRIX -A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal, that is, a square matrix $B = [b_{ij}]$ n × n is said to be a scalar matrix if $b_{ij} = 0$, when $i \neq j$ and $b_{ij} = k$, when $i = j$, for some constant k

IDENTITY MATRIX A square matrix in which elements in the diagonal elements are all equal to 1 and rest are all zero is called an identity matrix. In other words, the square matrix $A = [a_{ij}]$ n × n is an identity matrix if

$$a_{ij}=\begin{cases}1, i = j\\0, i \neq j\end{cases}$$

ZERO MATRIX -A matrix is said to be zero matrix or null matrix if all its elements are zero

EQUALITY OF TWO MATRICES - Two matrices A = $[a_{ij}]$ and B = $[b_{ij}]$ are said to be equal if (i) they are of the same order (ii) each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j

ADDITION OF MATRICES-The sum of two matrices is a matrix obtained by adding the corresponding elements of the given matrices. If A = $[a_{ij}]$ and B = $[b_{ij}]$ are two matrices of the same order, say m × n. Then, the sum of the two matrices A and B is defined as a matrix C = $[c_{ij}]$ m × n , where$c_{ij} = a_{ij} + b_{ij}$, for all possible values of i and j.

MULTIPLICATION OF A MATRIX BY A SCALAR- A= $[a_{ij}]$ m × n is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k.

DIFFERENCE OF TWO MATRICES- If A = $[a_{ij}]$, B = $[b_{ij}]$ are two matrices of the same order, say m × n, then difference A – B is defined as a matrix D = $[d_{ij}]$, where $d_{ij} = a_{ij} - b_{ij}$, for all value of i and j

PROPERTIES OF MATRIX ADDITION

1) Commutative Law - If A = $[a_{ij}]$, B = $[b_{ij}]$ are matrices of the same order, say m × n, then A + B = B + A.

2)Associative law- For any three matrices A = $[a_{ij}]$, B = $[b_{ij}]$, C = $[c_{ij}]$ of the same order, say m × n, (A + B) + C = A + (B + C)

3)Existence of additive identity- Let A = [aij] be an m × n matrix and O be an m × n zero matrix, then A + O = O + A = A. In other words, O is the additive identity for matrix addition.

4)The existence of additive inverse -Let A = $[a_{ij}]$ m × n be any matrix, then we have another matrix as – A =- $[a_{ij}]$ m × n such that A + (– A) = (– A) + A= O. Then -Ais the additive inverse of A or negative of A.

PROPERTIES OF SCALAR MULTIPLICATION OF A MATRX If $A = [a_{ij}]$ and $B = [b_{ij}]$ be two matrices of the same order, say $m \times n$, and k and l are scalars ,then

$$\text{(i)}\, k(A + B) = kA + kB, \quad \text{(ii)}\, (k + l)A = kA + lA$$

MULTIPLICATION OF TWO MATRICES-The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B. Let $A = [a_{ij}]$ be an $m \times n$ matrix and $B = [b_{jk}]$ be an $n \times p$ matrix. Then the product of the matrices A and B is the matrix C of order $m \times p$. To get the (i, k) th element $[C_{ik}]$of the matrix C, we take the i th row of A and kth column of B, multiply them elementwise and take the sum of all these products.

Non-commutativity of multiplication of matrices Even if AB and BA are both defined, it is not necessary that AB = BA

PROPERTIES OF MULTIPLICATION OF MATRICES

The associative law For any three matrices A, B and C. We have (AB) C = A (BC), whenever both sides of the equality are defined.

The existence of multiplicative identity For every square matrix A, there exist an identity matrix of same order such that IA = AI = A.

The distributive law For three matrices A, B and C. A (B+C) = AB + AC

(ii) (A+B) C = AC + BC

Transpose of a Matrix

If $A = [a_{ij}]$ be an $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A′ or (AT).

Properties of transpose of the matrices For any matrices A and B of suitable orders,(i) $(A')' = A$, (ii) $(A + B)' = A' + B'$ iii) $(A + B)' = A' + B'$ (iv) $(AB)' = B'A'$

Symmetric and Skew Symmetric Matrices)

A square matrix A = [a_{ij}] is said to be symmetric if A′ = A, that is, [a_{ij}] = [a_{ji}] for all possible values of i and j.

A square matrix A = [a_{ij}] is said to be skew symmetric matrix if A′ = – A, that is aji = – aij for all possible values of i and j.

Now, if we put i = j, we have a_{ij} = – aji. Therefore 2a_{ii} = 0 or a_{ii} = 0 for all i's. This means that all the diagonal elements of a skew symmetric matrix are zero.

For any square matrix A with real number entries, A + A′ is a symmetric matrix and A – A′ is a skew symmetric matrix

Any square matrix can be expressed as the sum of a symmetric and skew symmetrix and A – A′ is a skew symmetric matrix

Invertible Matrices If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the inverse matrix of A A and it is denoted by A^{-1}. In that case A is said to be invertible

MATRICES: MCQ QUESTIONS

1. If A is a square matrix such that $A^2 = I$, then$(A - I)^3 + (A+I)^3 - 7A$ is equal to

(a) A (b) I - A (c) I + A (d) 3A

2. If matrix A = $[a_{ij}]_{2\times2}$ where a_{ij} is equal to one , if $i \neq j$ and equal to zero if i= j then A^2 is equal to

(a) 0 (b) A (c) I (d) None of these

3.If A and B are square matrices such that $B = -A^{-1}BA$ then value of $(A + B)^2$ is

(a) $(A - B)^2$ (b) 0 (c) $A^2 + B^2$ (d) $A^2 - B^2$

4. Total number of possible matrices of order 3 x 3 with each entry 2 or 0 is

(a) 9 (b) 27 (c) 81 (d) 512

5. A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A' = I$ then α is

(a) π (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

6.If A is an invertible matrix and $A^{-1} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$ then A = ?

(a) $\begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{5} & \frac{1}{6} \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 2 \\ \frac{5}{2} & \frac{-3}{2} \end{bmatrix}$ (d) none of these

7.If A and B are 2x2 square matrices and A + B = $\begin{bmatrix} 4 & -3 \\ 1 & 6 \end{bmatrix}$and A - B = $\begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$

Then AB is

(a) $\begin{bmatrix} -7 & 5 \\ 1 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -5 \\ 1 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & -1 \\ 5 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & -1 \\ -5 & 5 \end{bmatrix}$

8.If A = $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ and $A^2 - xA + yI = 0$ then x + y is

(a) 10 (b) 23 (c) 19 (d) 17

9.If A = $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$ then

(a) a = 2 , b = - 3 (b) a = -2 , b = 3 (c) a = 1 , b = 4 (d) none of these

10.If A = $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ then value of $3A^2 - 2B + I$ is

(a) $\begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$ (b) **O** (c) $\begin{bmatrix} 4 & -20 \\ 18 & -15 \end{bmatrix}$ (d) none of these

11. If A $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ = $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ then A is equal to

(a) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

12.If $f(x) = x^2 + 4x - 5$ and A = $\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ then f(A) =

(a) $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$

13.Let A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ then A^n is equal to

(a) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a \end{bmatrix}$ (b) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ (c) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$ (d) $\begin{bmatrix} an & 0 & 0 \\ 0 & an & 0 \\ 0 & 0 & an \end{bmatrix}$

14. If $a_{ij} = \frac{1}{2}(3i + 2j)$ and A = $[a_{ij}]_{2x2}$ then $a_{21} + a_{22}$ is equal to

(a) 1 (b) 8 (c) 9 (d) -1

15.If A^I = $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ then $A^I - B^I$ IS

(a) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 2 & 3 \\ 0 & 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 3 \\ 3 & 0 \\ -1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -2 & 3 \\ 0 & 1 & -1 \end{bmatrix}$

16.For what values of k the matrix $\begin{bmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{bmatrix}$ is skew symmetric

(a) k = 1/2 (b) k = -3/2 (c) k = -1/2 (d) k = 3/2

17. Find x if $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

(a) 1 (b) 6 (c) - 9 (d) 13

18. Find a matrix A such that 2A – 3B + 5C = 0 where B = $\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$

and C = $\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

a) $\begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & -3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$ (c) $\begin{bmatrix} -8 & 3 & 5 \\ 13 & -1 & 9 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 3 & 5 \\ 13 & -1 & -9 \end{bmatrix}$

19. If A= $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ then value of (A- 2I) (A- 3I) is

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$

20. If A and B are symmetric matrices of same order ,then (AB^{I} - BA^{I}) is a

(a) symmetric matrix (b)skew symmetric matrix (c) null matrix (d) identity matrix

3 - MATRICES

ANSWER KEY MCQ QUESTIONS

1. (a) A 2. (c) I 3. (c) $A^2 + B^2$ 4. (d) 512 5. (b) $\frac{\pi}{3}$ 6. (c) $\begin{bmatrix} -3 & 2 \\ \frac{5}{2} & \frac{-3}{2} \end{bmatrix}$

7. (b) $\begin{bmatrix} 7 & -5 \\ 1 & 5 \end{bmatrix}$ 8. (b) 23 9. (c) a = 1 , b = 4

10. (a) $\begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$ 11. (c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ 12. (d) $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$

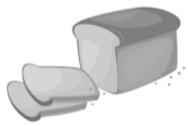
13. (c) $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$ 14. (c) 9 15. (a) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ 16. (b) k = -3/2

17. (d) 13 18. a) $\begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$ 19. (b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 20. (b)skew symmetric matrix

CASE STUDY QUESTIONS: MATRICES

1. Three people denoted by P1, P2, P3 intend to buy some rolls, buns, cakes and bread. Each of them needs these commodities in differing amounts and can buy them in two shops S1, S2. Which shop is the best for every person P1, P2, P3 to pay as little as possible? The individual prices and desired quantities of the commodities are given in the following tables:



Demanded quantity of foodstuff:

	roll	bun	cake	bread
P_1	6	5	3	1
P_2	3	6	2	2
P_3	3	4	3	1

Prices in shops S_1 and S_2 :

	S_1	S_2
roll	1.50	1.00
bun	2.00	2.50
cake	5.00	4.50
bread	16.00	17.00

Q1. The amount spent by the person P_1 in the shop S_1 is

a) 55
b) 50
c) 60
d) 62.5

Q2. The amount spent by person P_1 in the shop S_2 is

a) 49
b) 45
c) 48
d) 51

Q3. The cost of production for S2 if P1, P2 and P3 visit is 113.5. Find the profit earned by S2.

a) 38.5
b) 41.5
c) 40
d) None of the above

Q4. P_2 and P_3 are friends and decide to buy all the items from the shop S_2. How much will it cost them?

a) 53.5
b) 82
c) 104.5
d) None of the above

Q5. Which shop is the best for every person P_1, P_2, P_3 to pay as little as possible respectively?

a) $(P_1, P_2, P_3) : (S_1, S_1 \text{ or } S_2, S_2)$
b) $(P_1, P_2, P_3) : (S_2, S_2, S_2)$
c) $(P_1, P_2, P_3) : (S_1, S_2, S_1)$
d) $(P_1, P_2, P_3) : (S_2, S_1, S_1 \text{ or } S_2)$

2. Read the following and the answer the questions on the basis of the same.
Three schools A, B and C organized an event for collecting funds for helping the COVID effected victims. They sold toys, handmade boxes and notebook packets from the recycled material at a cost of 50, 30 and 40 each. The number of items sold is given below

ITEMS	A	B	C
TOYS	20	30	40
BOXES	50	40	50
NOTEBOOKS	40	20	30



Q1. Amount collected by school A is
a) 3500

b) 4100
c) 4500
d) 5100

Q2. Amount collected by school C is

a) 5700
b) 6000
c) 3500
d) 4700

Q3. The total amount raised by all the schools is

a) 12100
b) 12200
c) 12300
d) 12400

Q4. Total amount raised by toys is

a) 4000
b) 4500
c) 5000
d) 5500

Q5. School C collected ______ by selling notebooks

a) 1200
b) 1400
c) 1600
d) 1000

3. A manufacturer produces three products pen, pencil and compass which he sells in two markets. Annual sales are indicated below

MARKET	ITEMS (in numbers)		
	PEN	PENCIL	COMPASS
X	18000	2000	10000
Y	8000	20000	6000



If the unit sale price of pen, pencil and compass are 5, 2 and 3.5 respectively and unit cost of the above three items are 3, 1 and 2.5.

Q1. Total revenue of market X

a) 128000
b) 129000
c) 130000
d) None of the above

Q2. Total revenue of the market Y

a) 100000
b) 101000
c) 102000
d) None of the above

Q3. Total cost incurred in market X

a) 81000
b) 80500
c) 81500
d) 82000

Q4. Profits in the market X and Y are

a) (48000, 40000)
b) (48000, 44000)
c) (44000, 42000)
d) (48000, 42000)

Q5. Gross profit in both the markets

a) 90000
b) 86000
c) 92000
d) 88000

4. Three bike dealers X, Y and Z deals in 3 types of bikes cargo, folding and BMX bikes. The sales figure of 2020 and 2021 showed that dealer X sold 60 cargo, 25 folding and 5 BMX bikes in 2020 and 150 cargo, 75 folding, 10 BMX bikes in 2021. Dealer Y sold 50 cargo, 15 folding, 1 BMX bike in 2020 and 100 cargo, 25 folding and 3 BMX bikes in 2021. Dealer Z sold 45 cargo, 20 folding and 1 BMX bike in 2020 and 50 cargo, 30 folding and 5 BMX bikes in 2021.
Based on the above info, answer the following questions

Q1. The matrix summarizing the sales data of 2020 is

a) $\begin{bmatrix} 60 & 25 & 5 \\ 50 & 15 & 15 \\ 45 & 20 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 60 & 50 & 45 \\ 25 & 15 & 20 \\ 5 & 5 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 50 & 15 & 5 \\ 45 & 20 & 1 \\ 60 & 25 & 5 \end{bmatrix}$

d) $\begin{bmatrix} 45 & 20 & 1 \\ 50 & 15 & 5 \\ 60 & 25 & 5 \end{bmatrix}$

Q2. The matrix summarizing the sales data of 2021 is

a) $\begin{bmatrix} 150 & 100 & 50 \\ 75 & 25 & 30 \\ 10 & 3 & 5 \end{bmatrix}$

b) $\begin{bmatrix} 150 & 100 & 50 \\ 10 & 3 & 5 \\ 75 & 25 & 30 \end{bmatrix}$

c) $\begin{bmatrix} 150 & 75 & 10 \\ 100 & 25 & 3 \\ 50 & 30 & 5 \end{bmatrix}$

d) $\begin{bmatrix} 150 & 75 & 10 \\ 50 & 30 & 5 \\ 100 & 25 & 3 \end{bmatrix}$

Q3. The total number of bikes sold in two given years by each dealer is given by the matrix

a) $\begin{bmatrix} 210 & 150 & 95 \\ 100 & 40 & 50 \\ 15 & 8 & 6 \end{bmatrix}$

b) $\begin{bmatrix} 210 & 150 & 95 \\ 15 & 8 & 6 \\ 100 & 40 & 50 \end{bmatrix}$

c) $\begin{bmatrix} 100 & 40 & 50 \\ 15 & 8 & 6 \\ 210 & 150 & 95 \end{bmatrix}$

d) $\begin{bmatrix} 210 & 100 & 15 \\ 150 & 40 & 8 \\ 95 & 50 & 6 \end{bmatrix}$

Q4. The increase in sales in 2020 to 2021 is given by the matrix

a) $\begin{bmatrix} 90 & 50 & 5 \\ 50 & 10 & 10 \\ 5 & 2 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 90 & 50 & 5 \\ 5 & 2 & 4 \\ 50 & 10 & 10 \end{bmatrix}$

c) $\begin{bmatrix} 90 & 50 & 5 \\ 50 & 10 & 2 \\ 5 & 10 & 4 \end{bmatrix}$

d) $\begin{bmatrix} 50 & 10 & 10 \\ 90 & 50 & 5 \\ 5 & 2 & 4 \end{bmatrix}$

Q5. If each dealer receive the profit of 50000 on sale of a cargo, 100000 on sale of folding bike and 200000 on sale of a BMX. Then the amount of profit received in the year 2021 by each dealer is given by the matrix

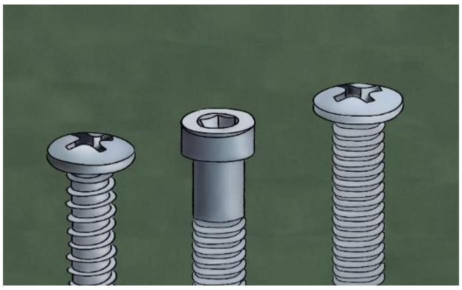
a) $\begin{pmatrix} 17000000 \\ 810000 \\ 6500000 \end{pmatrix}$

b) $\begin{pmatrix} 17000000 \\ 8100000 \\ 6500000 \end{pmatrix}$

c) $\begin{pmatrix} 1700000 \\ 8100000 \\ 6500000 \end{pmatrix}$

d) $\begin{pmatrix} 1700000 \\ 810000 \\ 650000 \end{pmatrix}$

5. A manufacturer produces three types of screws a, b, c which he sells in two firms. Monthly sales in rupees are given below
6.



FIRMS	Number of products per month		
	a	b	c
I	6000	20000	8000
II	10000	2000	18000

If unit sale prices of a, b and c are 2.5, 1.5, 1 respectively. Answer the following using matrices

Q1. Find the total monthly revenue collected from firm I

a) 49000
b) 51000
c) 53000
d) 55000

Q2. Find the total monthly revenue collected from firm II

a) 44000
b) 46000
c) 48000
d) 50000

Q3. If the unit cost of the above three items are 2, 1 and 50 paise respectively. Find the gross monthly profit from both the firms

a) 30000
b) 28000
c) 26000
d) 32000

Q4. What is annual sale of both the firms for screws of type: a, if the monthly sales remain constant throughout the year?

a) 3.2 lakhs
b) 4.8 lakhs
c) 5.6 lakhs
d) None

Q5.What is the total revenue generated by firm I from type b and type c screws in a month?

a) 36000
b) 38000
c) 40000
d) 42000

ANSWER KEY

	Q1	**Q2**	**Q3**	**Q4**	**Q5**
Case Study 1	b	a	c	c	d
Case Study 2	b	d	c	b	a
Case Study 3	b	b	a	c	b
Case Study 4	a	c	d	c	b

Case Study 5	c	b	d	b	b

MATRICES

ASSERTION-REASONING MCQs

DIRECTIONS: Each of these questions contains two statements: Assertion (A) and Reason (R). Each of these questions has four alternative choices ,any one of which is the correct answer .You have to select one of the codes (a), (b),(c) and (d) given below.

(a)Ais true, R is true: R is a correct explanation for A.

(b) Ais true, R is true:R is not a correct explanation for A.

(c)A is true:R is false.

(d)A is false :R is true.

Questions:

1.**Assertion (A)** If A=$\begin{bmatrix} 2 & 1+2i \\ 1-2i & 7 \end{bmatrix}$then $det(A)$ is real.

Reason (R). If A=$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$,$a_{ij}$ being complex numbers then $|A|$ is always real.

Answer: c

2.**Assertion (A)** If A=$\begin{bmatrix} 1 & 2 \\ 5 & -1 \end{bmatrix}$,then $|A|$=-11

Reason (R). If A=$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$,then $|A|$=$a_{11}a_{22} - a_{12}a_{21}$.

Answer: a

3. **Assertion (A)** If Δ=$\begin{vmatrix} 1 & 0 & 1 \\ 1 & -2 & 3 \\ 5 & 3 & 8 \end{vmatrix}$ then Δ =-12.

Reason (R) If we expand the determinant either by any row or any column , then the value of determinant always be same.

Answer: a

4. **Assertion (A)** If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$,then $x = \pm 6$.

Reason (R) If A and B are matrices of order 3 and $|A|$ =4,$|B|$ = 6, then $|2AB|$ =192.

Answer: b

5.**Assertion (A)** Determinant of a skew-symmetric matrix of order 3 is zero.

Reason (R) For any matrix A,$|A^T|$ = $|A|$ and $|-A|$ = -$|A|$

Answer: c

6. **Assertion (A)** The points A(a,b+c), B (b,c+a) and C(c,a+b) are collinear.

Reason (R) Area of a triangle with three collinear points is zero.

Answer: a

7. **Assertion (A)** The equation of the line joining A (1,3) and B (0,0) is given by y = 3x.

Reason (R) The area of triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) in the form of determinant is $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Answer: c

8. **Assertion (A)** $\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$,where A_{ij} is cofactor of a_{ij}.

Reason (R)Δ =Sum of the products of elements of any row (or column)with their corresponding cofactors.

Answer: a

9. **Assertion (A) The matrix A = $\begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{4} & \mathbf{8} \end{bmatrix}$ is singular.**

Reason (R)A square matrix A is said to be singular, if $|A|$ =0.

Answer: a

10. Assertion (A) If A =$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 5 & 2 \\ 3 & 7 \\ 1 & 2 \end{bmatrix}$,then both AB and BA are defined.

Reason (R) For the two matrices A and B, the product AB is defined if number of columns in A is equal to the number of rows in B.

Answer: a

11. **Assertion (A)** If A =$\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$,then A^{-1}=$\begin{bmatrix} 3 & -2 \\ 4 & 3 \end{bmatrix}$

Reason (R) If A =$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$,then A^{-1}=$\begin{bmatrix} \frac{2}{13} & \frac{-5}{13} \\ \frac{3}{13} & \frac{-1}{13} \end{bmatrix}$

Answer: d

UNIT: II

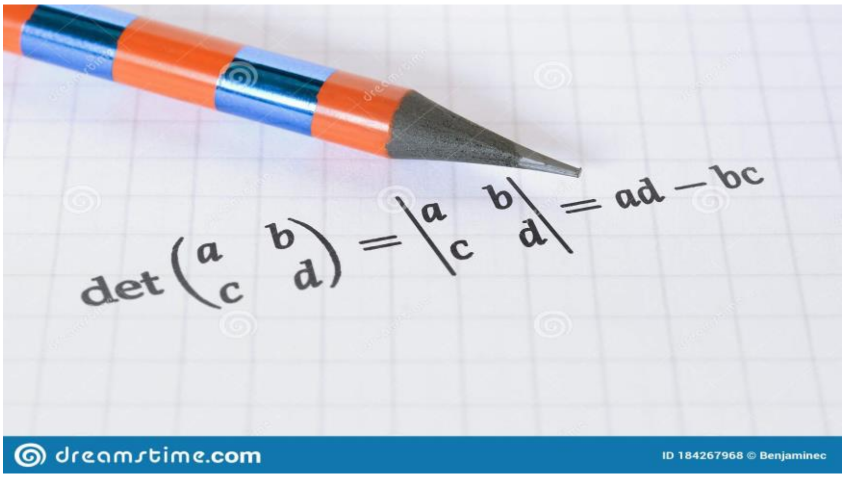
2.DETERMINANTS

1. GIST OF THE LESSON.

2. MULTIPLE CHOICE QUESTIONS.

3. CASE STUDY QUESTIONS.

4. ASSERTION REASONING QUESTIONS.



PART- 4

DETERMINANTS

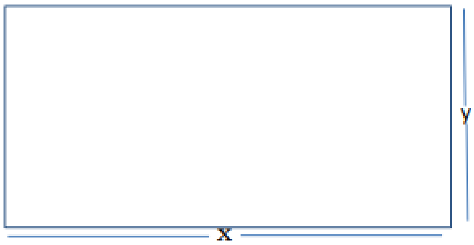
GIST OF THE LESSON CONTAINING POINTS TO REMEMBER AND FORMULA.
➢ To every square matrix we can assign a number called determinant.
➢ If A = [aij]. Then Det.A = IAI= $\|aij\|$ = aij
➢ If A= $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. Then IAI = $a_{11}a_{22} - a_{21}a_{12}$
➢ Area of the triangle having co-ordinates of vertices as (x_1,y_1) , (x_2, y_2) and (x_3,y_3) is $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$
➢ Points (x_1,y_1) , (x_2, y_2) and (x_3,y_3) are collinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$
➢ $\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$. where A_{ij} are cofactors of a_{ij} .
➢ If A and B are square matrix of order n, then $\|AB\| = \|A\|\|B\|$
➢ If A be a square matrix of order n, then $\|kA\| = k^n \|A\|$
➢ If A be a square matrix of order n, then $\|adj\ A\| = \|A\|^{n-1}$.
➢ If A and B are square matrices of the same order, then adj (AB)= adj B.adj A
➢ If A is a singular matrix, then $\|A\| = 0$
➢ If A is an invertible matrix, then $\|A\| \neq 0$ and $(A^{-1})^T = (A^T)^{-1}$
➢ If A be a square matrix of order n, then $\|adj(adjA)\| = \|A\|^{(n-1)^2}$
➢ If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then adj A $= \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$ (Note: to find adj(A) interchange diagonal elements and change the sign of non- diagonal elements.)
➢ If $\|A\| \neq 0$, then the system of equation is consistent and has a unique solution
➢ If A is a non-singular matrix , then $\|A^{-1}\| = \frac{1}{\text{IAI}}$
➢ If A is a non-singular matrix , then $\|(kA)^{-1}\| = \frac{1}{\text{k IAI}}$ ➢ For any square matrix A, A (Adj.A) = (Adj.A)A = $\|A\|l$

MULTIPLE CHOICE QUESTIONS -	
1	If A be a square matrix of order 2 and $\|A\| = 3$, then $\|5A\|$ is ____ a) 9 b) 75 c) 15 d) 2
2	If the points A (3, -2), B(k,2) and C (8,8) are collinear, then the value of k is: a) 2 b) -3 c) 5 d) -4
3	If A is a square matrix such that $A^2 = I$, then A^{-1} is equal to a) 2A b) O c) A d) A+I
4	The square matrix $A = \begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is a singular matrix, if the value of b is A) -3 b) 3 c) 0 d) arbitrary
5	Given that A is a square matrix of order 3 and $\| A \| = -4$, then $\| adj(A)\|$ is equal to: a) 4 b) -4 c) 16 d) -16
6	If A is a square matrix of order 4 such that $\|adj\ A\| = 125$, then $\|A\|$ is ____ a) 25 b) 5 c) 15 d) 625
7	Which of the following is a correct statement? a) Determinant is a square matrix b) Determinant is a number associated to a matrix c) Determinant is a number associated with the order of the matrix d) Determinant is a number associated to a square matrix
8	$If \begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x is equal to : a) 6 b) ±6 c) -1 d) -6
9	Find the cofactor of the element of second row and third column in the following det: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ a) 13 b) -13 c) 1 d) 2

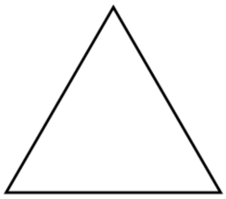
10	A = $\begin{bmatrix}3 & 4\\2 & 3\end{bmatrix}$, B = $\begin{bmatrix}-2 & -2\\0 & -1\end{bmatrix}$ Then $(A+B)^{-1}$ a) $\begin{bmatrix}-1 & 1\\1 & -1/2\end{bmatrix}$ b) does not exist c) $\begin{bmatrix}-1/2 & 1\\1 & -1\end{bmatrix}$ d) None of these
11	if A = $\begin{bmatrix}0 & 2\\3 & -4\end{bmatrix}$ and kA = $\begin{bmatrix}0 & 3a\\2b & 24\end{bmatrix}$, then the values of k,a and b respectively are : a) −6, −12, −18 b) −6, −4, −9 c) −6, 4, 9 d) −6, 12, 1
12	If A is a non-singular matrix of order 3 , then $\lvert(3A)^{-1}\rvert = \frac{1}{3\ IAI}$ a) $\frac{1}{3\ IAI}$ b) $\frac{1}{3^2\ IAI}$ c) $\frac{1}{3^3\ IAI}$ d) $\frac{1}{IAI}$
13	If B is a non- singular matrix and A is a square matrix, then $\lvert B^{-1}AB\rvert$ is a) $\lvert A^{-1}\rvert$ b) $\lvert B^{-1}\rvert$ c) $\lvert A\rvert$ d) $\lvert B\rvert$
14	If A = $\begin{bmatrix}a & 0 & 0\\0 & a & 0\\0 & 0 & a\end{bmatrix}$ then the value of $\lvert adj(A)\rvert$ is : a) a^3 b) a^{27} c) a^6 d) a^9
15	For any 2x2 matrix, A (AdjA) = $\begin{bmatrix}10 & 0\\0 & 10\end{bmatrix}$, then $\lvert A\rvert$ is equal to a) 20 b) 100 c) 10 d) 0
16	If A = $\begin{bmatrix}2 & 3\\5 & -2\end{bmatrix}$ be such that A^{-1} =kA, where k be a scalar then k is: a)19 b)1/19 c) -19 d) -1/19
17	The area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 sq. units. then k= a)$\pm$ 9 b) $\pm$3 c) -9 d) 3
18	If A be a square matrix of order 3, then $\lvert adj(adjA)\rvert$ = $a)\lvert A\rvert^6$ b) $\lvert A\rvert^2$ c) $\lvert A\rvert^4$ d) $\lvert A\rvert^3$
19	The value of $\begin{vmatrix}cos15^0 & sin15^0\\cos15^0 & sin15^0\end{vmatrix}$ a) 1 b) $\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) 0
20	If A = $\begin{bmatrix}1 & 2 & 2\\2 & 1 & x\\-2 & 2 & -1\end{bmatrix}$ is a matrix satisfying AA^T = 9 I, Then x = a) -2 b) 2 c) $\pm$2 d) 9

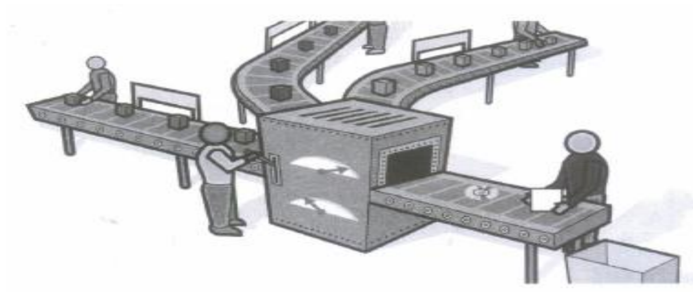
ANSWERS				
1) b	2) c	3) c	4) d	5) c
6) b	7) d	8) b	9) b	10) a
11) b	12) a	13) c	14) c	15) c
16) b	17) b	18) c	19) c	20) a

CASE STUDY TYPE QUESTIONS.

1	Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50m, then its area will remain same, but if length is decreased by 10m and breadth is decreased bv 20m. then its area will decrease by 5300 m^2  Based on the information given above, answer the following questions:
(i)	The equations in terms of x and y are : a) x-y=50, 2x-y=550 b) x-y=50, 2x+y=550 c) x + y = 50, 2x + y=550 d) x +y = 50, 2x - y=550
(ii)	Which of the following matrix equation is represented by the given information: a) $\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$ b) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$ c) $\begin{bmatrix}1 & 1\\2 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}50\\550\end{bmatrix}$ d) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-50\\-550\end{bmatrix}$
(iii)	The value of x (length of rectangular field) is : a) 150m b) 400m c) 200m d) 320m
(iv)	The value of y (breadth of rectangular field) is a) 150m b) 200m. c) 430m. d) 350m

(v)	The area of the rectangular field is: a) 60000Sq.m. b) 30000Sq.m. c) 30000m d) 3000m
2	Ram purchases 3 pens, 2 bags and 1 instrument box and pays ₹410/-. From the same shop Dheeraj purchases 2 pens, 1 bag and 2 instrument boxes and pays ₹290/-. While Ankur purchases 2 pens, 2 bags and 2 instrument boxes and pays ₹440/-.  Based on the information given above, answer the following questions:
(i)	Let $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}$ be a 3×3 matrix then $adj\ A$ is : a) $\begin{bmatrix} -2 & -2 & 3 \\ 0 & 4 & -4 \\ 2 & -2 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 0 & 2 \\ -2 & 4 & -2 \\ 3 & -4 & -1 \end{bmatrix}$ c) $\begin{bmatrix} 3 & -2 & -2 \\ -4 & 4 & 0 \\ -1 & -2 & 2 \end{bmatrix}$ d) None of these
(ii)	The cost of one pen is : a) ₹20 b) ₹50 c) ₹100 d) ₹150
(iii)	The cost of one pen and one bag is : a) ₹120 b) ₹150 c) ₹170 d) ₹250

(iv)	The cost of one pen and one instrument box is : a) ₹70 b) ₹120 c) ₹170 d) ₹250
(v)	The cost of one pen, one bag and one instrument box is : a) ₹220 b) ₹250 c) ₹200 d) ₹240
3	Area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by the determinant $\Delta = \frac{1}{2}\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ 3 & y_3 & 1 \end{bmatrix}$  Since, area is a positive quantity, so we always take the absolute value of the determinant A. Also, the area of the triangle formed by three collinear points is zero. Based on the above information, answer the following questions.
(i)	The area of the triangle whose vertices are (3, 8), (-4, 2), and (5, 1). a) 30 sq. units b) 35 sq. units c) 40 sq. units d) 30.5 sq. units
(ii)	If the points (2, -3), (k, -1) and (0, 4) are collinear, then the value of 4k is a) 4 b) $\frac{7}{40}$ c) $\frac{40}{7}$ d) 47
(iii)	If the area of a triangle ABC, with vertices A (1, 3), B (0, 0) and C (k, 0) is 3 sq. units, then the value of k is : a) 2 b) 3 c) 4 d) 5
(iv)	Using determinants, find the equation of the line joining the points A(1,2) & B(3,6). a) y = 2x b) x = 3y c) y = x d) 4x - y = 5
(v)	If A is (11, 7), B is(5, 5) and C is (-1, 3), then a) △ABC is scalene triangle b) A, B and C are collinear c) △ABC is equilateral triangle d) . None of these

4	Two schools A and B want to award their selected students on the values of Honesty, Hard work and Punctuality. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of Rs.2200. School B wants to spend Rs.3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school A). The total amount of award for one prize on each value is ₹1200. .  Using the concept of matrices and determinants, answer the following questions
(i)	What is the award money for Honesty? a) ₹350 b) ₹300 c) ₹500 d) ₹400
(ii)	What is the award money for Punctuality? a) Rs.300 b) Rs.280 c) Rs.450 d) Rs.500
(iii)	What is the award money for Hard work? a) Rs 500 b) Rs.400 c) 0 d) none of these
(iv)	If a matrix P is both symmetric and skew-symmetric, then IPI is equal to a) 1 b) -1 c) 0 d) none of these
(v)	If P and Q are two square matrices of same order such that PQ = Q and QP = P, then $\|Q^2\|$is equal to a) $\|Q\|$ b) $\|P\|$ c) 1 d) 0
5	A company produces three products every day. Their production on certain day is 45 tons. It is found that the production of third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product.  Using the concepts of matrices and determinants, answer the following questions.
(i)	If x, y and z respectively denotes the quantity (in tons) of first, second and third product produced, then which of the following is true? a) x + y + z = 45 b) x + 8 = z c) x-2y+z=0 d) all of these

(ii)	If $\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & -2\\ 1 & -1 & 1\end{bmatrix}^{-1} = \frac{1}{6}\begin{bmatrix}2 & 2 & 2\\ 3 & 0 & -3\\ 1 & -2 & 1\end{bmatrix}$, then inverse of $\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & -1\\ 1 & -2 & 1\end{bmatrix}$ is : a) $\begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{2} & 0 & \frac{-1}{2}\\ \frac{1}{6} & \frac{-1}{3} & \frac{1}{6}\end{bmatrix}$ b) $\begin{bmatrix}\frac{1}{2} & 0 & \frac{-1}{2}\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{6} & \frac{-1}{3} & \frac{1}{6}\end{bmatrix}$ c) $\begin{bmatrix}\frac{1}{3} & \frac{1}{2} & \frac{1}{6}\\ \frac{1}{3} & 0 & \frac{-1}{3}\\ \frac{1}{3} & \frac{-1}{2} & \frac{1}{6}\end{bmatrix}$ d) None of these
(iii)	x :y :z is equal to a) 12: 13: 20 b) 11:15:19 c) 15: 19: 11 d) 13:12:20
(iv)	Which of the following is not true? a) $\|A\| = \|A^T\|$ b) $[A^T]^{-1} = [A^{-1}]^T$ c) A is a skew symmetric matrix of odd order, then $\|A\| = 0$ d)$\|AB\| = \|A\| + \|B\|$
(v)	Which of the following is not true in the given determinant of A, where $A = [a_{ij}]_{3\times 3}$ a) Order of minor is less than order of det (A). b) Minor of an element can never be equal to cofactor of the same element c) Value of a determinant is obtained by multiplying elements of a row or column by corresponding cofactors. d) Order of minors and cofactors of the same elements of A is same.

ANSWERS

CS I	i. **b**	ii. **a**	iii. **c**	iv. **a**	v. **b**
CS II	i. **a**	ii. **a**	iii. **c**	iv. **a**	v. **a**
CS III	i. **d**	ii. **c**	iii. **a**	iv. **a**	v. **b**
CS IV	i. **b**	ii. **d**	iii. **b**	iv. **c**	v. **a**
CS V	i. **d**	ii. **c**	iii. **b**	iv. **d**	v. **b**

ASSERTION REASONING QUESTIONS

1	Assertion(A) : For a Matrix A= $[aij]_{4x4}$ if det(adjA) =1331 , then det(A) =11 Reason (R) : For a square Matrix A of order 'n' , | adj A | = $\|A\|^{n-1}$ (A) Both A & R are true and R is the correct explanation of A (B) Both A & R are true but R is not the correct explanation of A (C) A is true but R is false (D) A is false but R is true

2	Assertion (A) : For two square Matrices A & B of order3 , if |A|=4 & |B|=3 ,then |-5AB|=1500 Reason (R) : For asquare matrices A&B of order 'n' , and for a scalar 'k', |kA|=k^n|A| and |AB|=|A| |B| (A) Both A & R are true and R is the correct explanation of A (B) Both A & R are true but R is not the correct explanation of A (C) A is true but R is false D)A is false but R is true

ANSWERS

1	Explanation : If A is a square Matrix of order n , |adjA| =$\|A\|^{n-1}$ Here , A is of order 4 . Therefore, |adjA|=$\|A\|^3$=1331(given) ∴ |A|=11 ∴ Statement A is true Statement R is also true ∴**Correct Option is (A)**
2	Explanation : If A is a square Matrix of order n , |adjA| =$\|A\|^{n-1}$ Here , A is of order 4 . Therefore, |adjA|=$\|A\|^3$=1331(given) ∴ |A|=11 ∴ Statement A is true Statement R is also true ∴**Correct Option is (A)**

UNIT : III – CALCULUS

1. CONTINUITY AND DIFFERENTIABILITY

2.APPLICATIONS OF DERIVATIVES.

UNIT-III

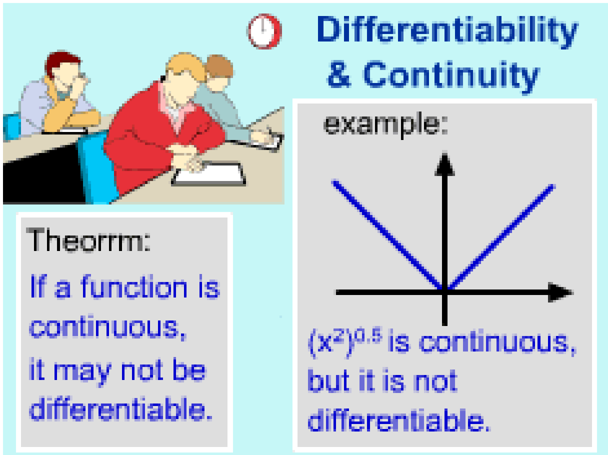
1. CONTINUITY AND DIFFERENTIABILITY.

1. GIST OF THE LESSON.

2. MCQ

3. CASE STUDY

4. ASSERTION AND REASONING



PART : 5

CONTINUITY AND DIFFERENTIABILITY.

GIST OF THE LESSON FOR QUICK REVISION

Continuous Function

A real valued function f is said to be continuous, if it is continuous at every point in the domain of f.

Continuity of a function at a point

Suppose f is a real valued function on a subset of real numbers and let c be a point in the domain of f, then f is continuous at x=c, if $\lim_{x \to c} f(x) = f(c)$

Ie., $\lim_{x \to c+} f(x) = \lim_{x \to c-} f(x) = f(c)$

Then f(x) is continuous at x=c . Otherwise f(x) is discontinuous at x=c

Graphically, a function f(x) is said to be continuous at a point if the graph of the function has no break either on the left or the right in the neighbourhood of the point.

Some basic continuous functions:

1. Every constant function is continuous.
2. Every identity function is continuous.
3. Rational functions are always continuous
4. Every polynomial function is continuous.
5. Modulus function f(x)= |x| is continuous.
6. All trigonometric functions are continuous in their domain.
7. e^x, logx continuous in their domain.

Algebra of continuous function

Theorem 1 :

Let f and g be two real functions, continuous at a real number c, then

1. (f + g) is continuous at x=c
2. (f-g) is continuous at x=c
3. fg is continuous at x=c
4. f/g is continuous at x=c provided g(c) ≠0

Differentiability

A real valued function f, is said to be differentiable at x=c in its domain, if its left hand and right hand derivatives at x=c exists and are equal.

At x=a, right hand derivative,

Rf'(a)=$\lim_{x\to 0}\left(\frac{f(a+h)-f(a)}{h}\right)$ and left hand derivative, Lf'(a) = $\lim_{x\to 0}\left(\frac{f(a-h)-f(a)}{-h}\right)$

Thus f(x) is differentiable at x=a , if Rf'(a) = Lf'(a)

Otherwise, f(x) is not differentiable at x=a

Derivatives of some standard functions:

1. $\frac{d}{dx}$(constant) =0
2. $\frac{d}{dx}(x^n)$ =nx^{n-1}
3. $\frac{d}{dx}$(sin x) = cos x
4. $\frac{d}{dx}$(cos x) = -sin x
5. $\frac{d}{dx}$(tan x) = sec^2x
6. $\frac{d}{dx}$(cosec x) = $-cosec\ x \cot x$
7. $\frac{d}{dx}$(sec x) = sec x tan x
8. $\frac{d}{dx}$(cot x) = $-cosec^2x$
9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(a^x) = a^x$ log a, a> 0
11. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$, x> 0
12. $\frac{d}{dx}(\log_a x) = \frac{1}{x\log_e a}$, a>0 , a ≠1

Algebra of derivatives

1. $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
2. $\frac{d}{dx}(uv) = u.\frac{dv}{dx} + v.\frac{du}{dx}$ (product rule)
3. $\frac{d}{dx}(\frac{u}{v}) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$ (quotient rule) where u and v are functions of x
4. $\frac{d(ku)}{dx} = k.\frac{du}{dx}$ where k is a constant

Chain Rule

Example 1:

If y = $\sin(x^2)$, then $\frac{dy}{dx} = \cos(x^2).\frac{d}{dx}(x^2) = \cos(x^2).2x = 2x\cos(x^2)$.

Example 2:

If y= tan(2x+3) , then $\frac{dy}{dx} = sec^2(2x+3).\frac{d}{dx}(2x+3)$

$$= sec^2(2x+3).2 = 2sec^2(2x+3).$$

Example 3 :

If y= $\sin(\cos(x^2))$, then $\frac{dy}{dx} = \cos(\cos(x^2)).\frac{d(\cos(x^2))}{dx}$

$$= \cos\left(\cos(x^2)\right). - \sin(x^2).\frac{d}{dx}(x^2)$$

$$= \cos\left(\cos(x^2)\right). - \sin(x^2).2x$$

$$= -2x.\sin(x^2)\cos\left(\cos(x^2)\right)$$

Derivative of implicit function

Let f(x,y) = 0 be an implicit function of x, then, to find $\frac{dy}{dx}$, first differentiate both sides of equation w.r.t x and then take all terms involving $\frac{dy}{dx}$ to LHS and remaining terms to RHS , then find $\frac{dy}{dx}$.

Derivatives of inverse trigonometric functions

1. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$, -1 < x< 1
2. $\frac{d}{dx}(cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$, -1 < x< 1
3. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$
4. $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$
5. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x.\sqrt{x^2-1}}$, |x| > 1
6. $\frac{d}{dx}(\text{cosec}^{-1}x) = \frac{-1}{x.\sqrt{x^2-1}}$, |x| > 1

Derivative of a function w.r.t another function

Let u = f(x) and v= g(x) be two given functions. Differentiate both functions w.r.t x separately and substitute in the following formula.

$$\frac{du}{dv} = \frac{du}{dx} \div \frac{dv}{dx}$$

Derivative of logarithmic functions

Suppose, given function is of the form u(x) $^{v(x)}$

In such cases, take logarithm on both sides and use properties of logarithm to simplify it. Then differentiate it.

Derivatives of parametric functions

If x= f(t) and y = g(t), then

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

Second order derivative

Let y=f(x) be a given function, then $\frac{dy}{dx}$ is called first derivative of y.

$\frac{d}{dx}(\frac{dy}{dx})$ is called the second order derivative of y w.r.t x and is denoted by

$\frac{d^2y}{dx^2}$ or y'' or y_2

MCQ ON CONTINUITY AND DIFFERENTIABILITY

1. If the function $f(x) = \begin{cases} \frac{(x+3)^2-36}{x-3}, & x \neq 3 \\ k & , x = 3 \end{cases}$ is given to be continuous at x= 3, then the value of k is
   a. 6 b. 12 c. -12 d. -6
2. If the function $f(x) = \begin{cases} \frac{k\cos x}{\pi-2x}, & x \neq \frac{\pi}{2} \\ 3 & , x = \frac{\pi}{2} \end{cases}$ is given to be continuous at $x = \frac{\pi}{2}$, then the value of k equals to
   a. 6 b. -6 c. 5 d. -5
3. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ when $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$
   a. $\frac{2x}{\sqrt{1-x^2}}$ b. $\frac{1}{2\sqrt{1-x^2}}$ c. $\frac{2}{\sqrt{1-x^2}}$ d. $\frac{1}{\sqrt{1-x^2}}$
4. If $x = \sqrt{a^{\sin^{-1}t}}$ *and* $y = \sqrt{a^{\cos^{-1}t}}$ then $\frac{dy}{dx}$ =?
   a. $\frac{y}{x}$ b. $-\frac{y}{x}$ c. $\frac{x}{y}$ d. $-\frac{x}{y}$
5. The function $f: R \to R$ is given by $f(x) = -|x-10|$ is
   a. Continuous as well as differentiable at x = 10
   b. Not continuous but differentiable at x = 10
   c. Continuous but not differentiable at x = 10
   d. Neither continuous nor differentiable at x = 10
6. If $y = \log\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ $at\ x = \frac{\pi}{4}$ is
   a. 0 b. 1 c. $\frac{1}{2}$ d. ∞

7. If $y = x + \sqrt{x^2 + a^2}$, then $\frac{dy}{dx}$ is equal to
   a. $\frac{y}{\sqrt{x^2+a^2}}$ b. $\frac{x}{\sqrt{x^2+a^2}}$ c. $\frac{x}{\sqrt{x+a}}$ d. $\frac{y}{\sqrt{x+a}}$

8. If $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$ and $y = a \sin t$, then $\frac{dy}{dx}$ is equal to
   a. cot t b.tan t c. sec t d. cosec t

9. If $y = 3\cos x + 3\sin x$, then $\frac{d^2y}{dx^2} + y$ is equal to
   a. 0 b. 1 c. 2 d. 3

10. If $f(x) = x^2 \sin\frac{1}{x}$, where x ≠ 0, then the value of the function f at x = 0, so that the function is continuous at x=0 is
    a. 0 b. -1 c. 1 d. None of the above

11. The number of points of discontinuity of the function f defined by $f(x) = |x| - |x+1|$ is
    a. 0 b. 1 c. 2 d. 3

12. If $x = a\sec^3\theta$ *and* $y = a\tan^3\theta$, then $\frac{dy}{dx}$ *at* $\theta = \frac{\pi}{3}$ is
    a. 1 b. $\frac{1}{2}$ c. $\frac{1}{\sqrt{3}}$ d. $\frac{\sqrt{3}}{2}$

13. If $y = (\tan^{-1} x)^2$, then the value of $(x^2+1)^2 y_2 + 2x\,(x^2+1)y_1$ is
    a. 2 b. 3 c. 4 d. None of the above

14. If $y = \log\left(\frac{x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to
    a. $\frac{1}{x\,(1+x^2)}$ b. $\frac{2x}{(1+x^2)}$ c. $\frac{2}{x\,(1+x^2)}$ d. $\frac{x}{(1+x^2)}$

15. 2x+3y = sin y , then $\frac{dy}{dx}$ is equal to
    a. $\frac{2}{\cos y}$ b. $\frac{2}{\cos y+3}$ c. $\frac{2}{\cos y-3}$ d. $\frac{2}{3-\cos y}$

16. Find the value of ‘a’ for which the function f defined as
    $$f(x) = \begin{cases} a\sin\frac{\pi}{2}(x+1) & , \ x \le 0 \\ \frac{\tan x - \sin x}{x^3} & , \ x > 0 \end{cases}$$ is continuous at x= 0
    a. $\frac{\pi}{2}$ b. $-\frac{\pi}{2}$ c. $\frac{1}{2}$ d. $-\frac{1}{2}$

17. Derivative of $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, $0 < x < \frac{\pi}{2}$ is
    a. $\frac{1}{2}$ b. 1 c. 2 d. None of the above

18. If $x = 2at$ and $y = at^2$, where a is a constant then $\frac{d^2y}{dx^2}$ at $x = \frac{1}{2}$ is

a. $\frac{1}{2a}$ b. 1 c. 2a d. $\frac{a}{2}$

19. The derivative of $\cos^{-1}(2x^2 - 1)$ wrt $\cos^{-1} x$ is

a. 2 b. $-\frac{1}{2\sqrt{1-x^2}}$ c. $\frac{2}{x}$ d. $1 - x^2$

20. If $f(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sqrt{1+bx}-1}{x}, & x > 0 \end{cases}$ is continuous at x=0, then find the values of a & b

a. a=0 & b=5 b. a= -1 & b=4 c. a= -2& b=3 d. a= -3& b=2

CASE-STUDY

Q1) Read the following text and answer the following questions on the basis of the same:

Mr. Rahul of Allien School is teaching **chain rule** to his students with the help of a flow –chart

The chain rule says that if h and g are functions and $f(x) = [g(h(x)]$, *then*

$f'(x) = [g(h(x)]' = g'(h(x).h'(x)$

21. The derivative of $e^{\sin^{-1} x}$ is

A) $\frac{e^{\sin^{-1} x}}{\sqrt{1+x^2}}$ (B) $\frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}$ (c) $e^{\sin^{-1} x}(\sqrt{1-x^2})$ (D) $x\,\frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}$

22. Differentiate $\sqrt{\cos\sqrt{x}}$ w. r .t. x.

(A) $-\frac{\sin\sqrt{x}}{2\sqrt{\cos\sqrt{x}}}$ (B) $\frac{\sin\sqrt{x}}{\sqrt{\cos\sqrt{x}}}$ (C) $-\frac{\sin\sqrt{x}}{4\sqrt{\cos\sqrt{x}}}$ (D) $\frac{\sin\sqrt{x}}{2\sqrt{\cos\sqrt{x}}}$

23. Derivative of $\log\left(\frac{a+x}{a-x}\right)$ with respect to x is

(A) $\frac{a+x}{a-x}$ (B) $\frac{2x}{a-x}$ (C) $\frac{a-x}{a+x}$ (D) $\frac{2a}{a^2-x^2}$

24. $\frac{d}{dx}[\sin(x^x)]$ *is*

(A) $x^x(1+logx)$ (B) $cosx.x^x(1+logx)$

(C) $\cos(x^x)(1+logx)$ (D) $x^x\cos(x^x)(1+logx)$

25. If $y = \log\tan\frac{x}{2}$ then $\frac{dy}{dx}$ is

A) cosx (B) s ec x C) cosec x (D)tan x

ASSERTION & REASON TYPE

In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is true

26. If $f(x) = \begin{cases} mx+1 & \text{if } x \le \frac{\pi}{2} \\ \sin x + n & \text{if } x > \frac{\pi}{2} \end{cases}$, is continous at at $x = \frac{\pi}{2}$, then

Assertion (A): The relation between m and n is $m\pi = 2n$

Reason(R): f(x) is continuous at x = a , if $lim_{x\to a-}f(x) = lim_{x\to a+}f(x) = f(a)$

27. **Assertion (A):** The differential coefficient of

$$\mathbf{cosec^{-1}\frac{1}{2x^2-1}\ \textit{with respect to}\ \sqrt{1-x^2}\ \textit{at}\ x=\frac{1}{2}\ \textit{is}\ 4}$$

Reason(R): The derivative of u with respect to v is $\frac{du/dx}{dv/dx}$

28. **Assertion (A):** If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is $\frac{\log x}{(1+\log x)^2}$

Reason(R): If $\mathbf{y} = \mathbf{x}^{\mathbf{f(x)}}$, then $\frac{dy}{dx} = \mathbf{x}^{\mathbf{f(x)}}\ [\ f'(x)\ \log x + \frac{f(x)}{x}\ ]$

29. **Assertion (A):** Let $x = \cos\theta + \theta\sin\theta$, $y = \sin\theta - \theta\cos\theta$, where θ is the parameter then $\frac{d^2y}{dx^2} = \frac{\sec^3\theta}{\theta}$

Reason(R): $\frac{d^2y}{dx^2} = \frac{d^2y}{d\theta^2} \div \frac{d^2x}{d\theta^2}$

30. **Assertion (A):** $If\ y = \tan^{-1}\left(\frac{cosx+sinx}{cosx-sinx}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$ $, then \frac{dy}{dx} = -1$

Reason(R): $\frac{cosx+sinx}{cosx-sinx} = \tan(\frac{\pi}{4} + x)$

Answer Key

1.b	2.a	3.c	4.b	5.c	6.b	7.a	8.b	9.a	10.a
11.a	12.d	13.a	14.c	15.c	16.c	17.a	18.a	19.a	20.b
21.B	22.C	23.D	24.D	25.C	26.A	27.D	28.B	29.C	30.D

UNIT: III

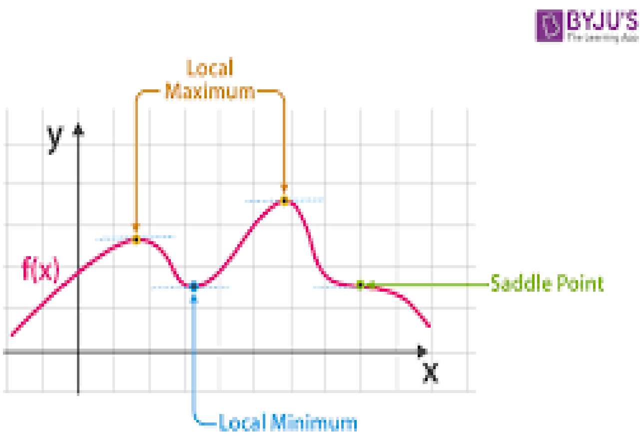
2.APPLICATIONS OF DERIVATIVES.

1.Gist of the lesson

2. MCQs

3.Case Study Based Questions

4. Assertion and Reasoning



PART-6

APPLICATION OF DERIVATIVES

KEY POINTS:--

- If two variables x and y are varying with respect to another variable t, i.e., if $x = f(t)$ and $y= g(t)$, then by **Chain Rule** $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$, if $\frac{dx}{dt} \neq 0$.

- A function f is said to be
  (a) **increasing** on an interval (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in$ (a, b).
  Alternatively, if $f'(x) \geq 0$ for each x in (a, b)
  (b) **strictly increasing** on an interval (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ (a, b).
  Alternatively, if $f'(x) > 0$ for each x in (a, b)
  (c) **decreasing** on (a,b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in$ (a, b).
  Alternatively, if $f'(x) \leq 0$ for each x in (a, b)
  (d) **strictly decreasing** on (a,b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in$ (a, b).
  Alternatively, if $f'(x)<0$ for each x in (a, b)
  (e) **constant** in (a, b), if $f(x) = c$ for all $x \in$ (a, b), where c is a constant. Then $f'(x)=0$.

- **The equation of the tangent at (x_0, y_0)** to the curve $y = f(x)$ is given by
  $$y - y_o = \left.\frac{dy}{dx}\right]_{(x_0,y_o)} (x - x_o)$$
- **Equation of the normal** to the curve $y = f(x)$ at a point (x_0, y_0) is given by
  $$y - y_o = \frac{-1}{\left.\frac{dy}{dx}\right]_{(x_0,y_o)}} (x - x_o)$$
- A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called a critical point of f.

- **First Derivative Test:--** Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then
  (i) If $f'(x)$ changes sign from positive to negative as x increases through c, i.e., if $f'(x) > 0$ at every point sufficiently close to and to the left of c, and $f'(x) < 0$ at every point sufficiently close to and to the right of c, then c is a point of local maxima.
  (ii) If $f'(x)$ changes sign from negative to positive as x increases through c, i.e., if $f'(x) < 0$ at every point sufficiently close to and to the left of c, and $f'(x) > 0$

at every point sufficiently close to and to the right of c, then c is a point of local minima.

(iii) If f ′(x) does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. In fact, such a point is called point of inflexion.

- **<u>Second Derivative Test:--</u>** Let f be a function defined on an interval I and c ∈ I. Let f be twice differentiable at c. Then

  (i) x = c is a point of local maxima if f ′(c) = 0 and f ″(c) < 0 The values f (c) is local maximum value of f .

  (ii) x = c is a point of local minima if f ′(c) = 0 and f ″(c) > 0 In this case, f (c) is local minimum value of f . (iii) The test fails if f ′(c) = 0 and f ″(c) = 0. In this case, we go back to the first derivative test and find whether c is a point of maxima, minima or a point of inflexion.

- **<u>Working rule for finding absolute maxima and/or absolute minima</u>**

  Step 1: Find all critical points of f in the interval, i.e., find points x where either f ′(x) = 0 or f is not differentiable.

  Step 2: Take the end points of the interval.

  Step 3: At all these points (listed in Step 1 and 2), calculate the values of f .

  Step 4: Identify the maximum and minimum values of f out of the values calculated in Step 3. This maximum value will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f .

<u>MCQ QUESTIONS</u>

1) The function f(x) $= \tan x - x$

(a) always increases (b) always decreases (c) never increases (d) none of these

2) The equation of tangent at (2,3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$, then the value of $a - b$

(a) 9 (b) -5 (c) -9 (d) 5

3) The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line $x + 3y = 8$ is

(a) $3x - y = 8$ (b) $3x + y + 8 = 0$

(c) $x + 3y \pm 8 = 0$ (d) $x + 3y = 0$

4) A missile is fired from the ground level rises x metres vertically upwards in t seconds where

$x = 100t - \frac{25}{2}t^2$. The maximum height reached is

(a) 125 m (b) 200 m (c) 190 m (d) 300 m

5) The minimum value of $y = x^4 + 1$ is

(a) -1 (b) 1 (c) 0 (d) not defined

6) The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is

(a) $\frac{1}{3}$ (b) 3 (c) -3 (d) $-\frac{1}{3}$

7) The point on the curve $y = (x-3)^2$, where the tangent is parallel to the chord joining (3,0) and (4,1) is

(a) $\left(-\frac{7}{2}, \frac{1}{4}\right)$ (b) $\left(\frac{5}{2}, \frac{1}{4}\right)$ (c) $\left(-\frac{5}{2}, \frac{1}{4}\right)$ (d) $\left(\frac{7}{2}, \frac{1}{4}\right)$

8) If $f(x) = x + 2,\ x \in (0,1)$, then

(a) the function f has not a local maximum value

(b) the function f has not a local minimum value

(c) Both (a) and (b) are true

(d) Bothe (a) and (b) are false

9) The coordinates of the point on the curve $\sqrt{x} + \sqrt{y} = 4$ at which the tangent is equally inclined to the axis is

(a) (4,4) (b) (2,4) (c) (3,4) (d) (2,2)

10) Equation of line passing through a point (1,4) and the sum of the intercept on the positive axis is

minimum is

(a) $2x + y - 6 = 0$ (b) $x + 2y - 9 = 0$ (c) $y + 2x + 6 = 0$ (d) none of these

11) If $x + y = K$ is normal to $y^2 = 12x$, then K is

(a) 3 (b) 9 (c) -9 (d) -3

12) The measure of angle between the curves $y^2 = 32x$ and $x^2 = 108y$ at (0,0) is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

13) The curve $y = xe^x$ has minimum value equal to

(a) $-\frac{1}{e}$ (b) $\frac{1}{e}$ (c) $-e$ (d) e

14) The distance between the origin and the normal to the curve $y = e^{2x} + x^2$ at $x = 0$ is

(a) 2 units (b) $\frac{\sqrt{3}}{2}$ units (c) $\frac{\sqrt{5}}{2}$ (d) $\frac{2}{\sqrt{5}}$

15) If $f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$, then which of the following holds?

I . $f(x)$ has local maxima at $x = 2$

II. $f(x)$ has local minima at $x = 5$

III. $f(x)$ has local minima at $x = -3$

(a) Only I is true (b) Only III is true

(c) Both I and II are true (d) All I, II and III are true

16) If $y = x(x-3)^2$ decreases for the values of x given by

(a) $1 < x < 3$ (b) $x < 0$ (c) $x > 0$ (d) $0 < x < \frac{3}{2}$

17) If the curve $ay + x^2 = 7$ and $x^3 = y$, cut orthogonally at (1,1), then the value of a is

(a) 1 (b) 0 (c) -6 (d) 6

18) The minimum value of $2x + 3y$ when $xy = 6$ is

(a) 9 (b) 12 (c) 18 (d) 6

19) The maximum value of $f(x) = (x-1)^{\frac{1}{3}}(x-2)$ in [1,9] is

(a) 9 (b) 0 (c) 14 (d) $\frac{5}{4}$

20) Equation of tangent at the curve $y = be^{\frac{-x}{a}}$, where it crosses the Y-axis is

(a) $\frac{x}{a} + \frac{y}{b} = 1$ (b) $\frac{x}{a} - \frac{y}{b} = 1$

(c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (d) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

ANSWER KEY

1. (a)
2. (a)
3. (c)
4. (b)
5. (b)
6. (d)
7. (d)
8. (c)
9. (a)
10. (a)
11. (b)
12. (b)
13. (a)
14. (d)
15. (b)
16. (a)
17. (d)
18. (b)
19. (c)
20. (a)

SOLUTIONS

1) $f(x) = \tan x - x$

$f'(x) = \sec^2 x - 1$, where $\sec x > 1$ *or* $\sec x < 1$ so that $\sec^2 x > 1\ \forall x$

Hence $f'(x) > 0\ \forall\ x$

$f(x) = \tan x - x$ increases always

2) $y^2 = ax^3 + b$

$2y\frac{dy}{dx} = 3ax^2$

$\frac{dy}{dx} = \frac{3ax^2}{2y}$

Slope of tangent at (2,3) = $\frac{3a\times2^2}{2\times3} = 2a$

Equation of tangent is $y = 4x - 5\ (given)$. So slope of the tangent = 4

$\therefore 2a = 4$, then $a = 2$

$(2,3)$ lies on the curve so that $3^2 = 2 \times 2^3 + b$, then $b = -7$

Hence $a - b = 2 - (-7) = 9$

3) Let the point on the curve $3x^2 - y^2 = 8$ at which the normal drawn is (a, b)

Then $3a^2 - b^2 = 8 \ldots\ldots\ldots (1)$

$3x^2 - y^2 = 8 \Longrightarrow 6x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{3x}{y} \Longrightarrow$ slope of the normal = $(-\frac{y}{3x})_{(a,b)} = -\frac{b}{3a}$

Slope of the given line = $-\frac{1}{3}$

$-\frac{b}{3a} = -\frac{1}{3}$ (*normal is parallel to the line*)

Hence $b = a$

Equation (1) becomes $3a^2 - a^2 = 8 \Longrightarrow 2a^2 = 8 \Longrightarrow a^2 = 4$ hence $a = \pm 2$

So $a = 2, b = 2$ or $a = -2$ or $b = -2$

At (2,2) the equation of normal is $y - 2 = -\frac{1}{3}(x - 2)$. Ie $x + 3y - 8 = 0$

At $(-2, -2)$ the equation of normal is $y + 2 = -\frac{1}{3}(x + 2)$. Ie $x + 3y + 8 = 0$

Hence the equation of the normal is $x + 3y \pm 8 = 0$

4) $x = 100t - \frac{25}{2}t^2$

$\frac{dx}{dt} = 100 - 25t$

$100 - 25t = 0 \Longrightarrow t = 4$

$\frac{d^2x}{dt^2} = -25 < 0$ so that x is maximum when $t = 4$

Hence maximum height = $100 \times 4 - \frac{25}{2} \times 4^2 = 200$

5) $y = x^4 + 1$ where $x^4 \geq 0 \ \ \forall x$

So $x^4 + 1 \geq 1$

6) $y = 2x^2 + 3\sin x$

$\frac{dy}{dx} = 4x + 3\cos x$, a $x = 0$ $\frac{dy}{dx} = 0 + 3\cos 0 = 3$

Slope of the normal = $-\frac{1}{3}$

7) Let the point be (a, b) on $y = (x-3)^2$ so that $b = (a-3)^2$

$y = (x-3)^2$

$\frac{dy}{dx} = 2(x-3)$, slope of the tangent = $2(a-3)$

Slope of the given chord = $\frac{1-0}{4-3} = 1$

$2(a-3) = 1(tangent\ is\ parallel\ to\ the\ chord)$

$a = \frac{7}{2}$ then $b = (\frac{7}{2} - 3)^2 = \frac{1}{4}$

Hence the required point is $\left(\frac{7}{2}, \frac{1}{4}\right)$

8) $f(x) = x + 2$

$f'(x) = 1 \ \ \forall x$, so for any value of x $f'(x)$ cannot be 0. So $f(x)$ has no critical points.

Hence $f(x)$ has neither local maximum not local minimum

9) Let the point on the curve $\sqrt{x} + \sqrt{y} = 4$ be (a, b) so that $\sqrt{a} + \sqrt{b} = 4$ …..(1)

$\sqrt{x} + \sqrt{y} = 4$

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$, then $\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

Slope of the tangent = $-\frac{\sqrt{b}}{\sqrt{a}}$.

Slope of the tangent = tan 45 or tan 135(given) = ± 1

$-\frac{\sqrt{b}}{\sqrt{a}} = \pm 1 \implies \frac{b}{a} = 1 \implies a = b$

Equation (1) becomes $\sqrt{a} + \sqrt{a} = 4$

$$2\sqrt{a} = 4\ or\ \sqrt{a} = 2$$

Hence $a = 2^2 = 4$ then $b = 4$ So the required point is (4,4)

10) Let the equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$

(1,4) lies on the line so that $\frac{1}{a} + \frac{4}{b} = 1$

$$\frac{4}{b} = 1 - \frac{1}{a} = \frac{a-1}{a}$$

$$b = \frac{4a}{a-1}$$

Sum of the intercepts $= a + b = a + \frac{4a}{a-1} = \frac{a^2-a+4a}{a-1} = \frac{a^2+3a}{a-1}$

Let $f(a) = \frac{a^2+3a}{a-1}$

$$f'(a) = \frac{(a-1)(2a+3)-(a^2+3a)(1)}{(a-1)^2} = \frac{2a^2+3a-2a-3-a^2-3a}{(a-1)^2} = \frac{a^2-2a-3}{(a-1)^2}$$

$$f'(a) = 0 \Longrightarrow \frac{a^2-2a-3}{(a-1)^2} = 0$$

$a^2 - 2a - 3 = 0 \Longrightarrow a = -2, 3$ (but given intercepts on positive axes)

So $x = 3$ then $b = \frac{4\times3}{3-1} = 6$

Hence the equation is $\frac{x}{3} + \frac{y}{6} = 1$ or $2x + y - 6 = 0$

11) $y^2 = 12x$

$2y\frac{dy}{dx} = 12$

$\frac{dy}{dx} = \frac{6}{y}$ So slope of the normal = $-\frac{y}{6}$

Given normal $x + y = K$ so that slope = -1

$-\frac{y}{6} = -1 \Longrightarrow y = 6$ then $12x = 6^2$

$$x = 3$$

$K = x + y = 3 + 6 = 9$

12) $y^2 = 32x$

$2y\frac{dy}{dx} = 32$

$\frac{dy}{dx} = \frac{16}{y}$ then the slope of the tangent at (0,0) $= \tan\alpha = \frac{16}{0} \Longrightarrow \alpha = \frac{\pi}{2}$

$x^2 = 108y$

$2x = 108\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{x}{54}$ then the slope of the tangent at (0,0) = $\tan\beta = \frac{0}{54} \Longrightarrow \beta = 0$

Hence the angle between the curves = the angle between the tangents to the curves at (0,0)

$$=|\alpha - \beta|$$

$$= \frac{\pi}{2}$$

13) $y = xe^x$

$\frac{dy}{dx} = xe^x + e^x = e^x(x+1)$

$\frac{dy}{dx} = 0 \Longrightarrow e^x(x+1) = 0 \Longrightarrow x = -1$

$\frac{d^2y}{dx^2} = e^x + (x+1)e^x = e^x(x+2)$

at $x = -1, \frac{d^2y}{dx^2} = e^{-1} > 0$

So y is minimum at $x = -1$ and the minimum value of $y = (-1)e^{-1} = -\frac{1}{e}$

14) $y = e^{2x} + x^2$

$\frac{dy}{dx} = 2e^{2x} + 2x$. At $x = 0$, $\frac{dy}{dx} = 2e^0 + 2 \times 0 = 2$

Then slope of the normal = $-\frac{1}{2}$

When $x = 0,\ y = e^0 + 0 = 1$

Equation of the normal is $y - 1 = -\frac{1}{2}(x - 0)$

$x + 2y - 2 = 0$

Distance of the normal from the origin = $\left|\frac{-2}{\sqrt{1^2+2^2}}\right| = \frac{2}{\sqrt{5}}$

15) $f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$

$f'(x) = -\frac{3}{4} \times 4x^3 - 8 \times 3x^2 - \frac{45}{2} \times 2x$

$= -3x^3 - 24x^2 - 45x$

$= -3x(x^2 + 8x + 15)$

$= -3x(x+3)(x+5)$

$f''(x) = -9x^2 - 48x - 45$

$f'(x) = 0 \Longrightarrow -3x(x+3)(x+5) = 0 \Longrightarrow x = 0, x = -3, x = -5$

$f''(0)=-45<0$, $f''(-3)=18>0$, $f''(-5) = -30 < 0$

So $f(x)$ is minimum at $x = -3$

16) $y = x(x-3)^2$

$\frac{dy}{dx} = x \times 2(x-3) + (x-3)^2$

$= (x-3)(2x + x - 3) = (x-3)(3x-3) = 3(x-3)(x-1)$

$\frac{dy}{dx} = 0 \Longrightarrow x = 3, x = 1$

f’(x)	$(-\infty, 1)$	$(1,3)$	$(3, \infty)$
$x-3$	$-$	$-$	$+$
$x-1$	$-$	$+$	$+$
3	$+$	$+$	$+$
$f'(x)$	$+$	$-$	$+$

$f(x)$ is decreasing in $(1,3)$

17) $ay + x^2 = 7$

$a\frac{dy}{dx} + 2x = 0$

$\frac{dy}{dx} = \frac{-2x}{a}$

Slope of tangent to the 1st curve, $m_1 = \frac{dy}{dx}\ at\ (1,1) = \frac{-2}{a}$

$x^3 = y$

$3x^2 = \frac{dy}{dx}$

Slope of tangent to the 2nd curve , $m_2 = \frac{dy}{dx}\ at\ (1,1) = 3(1)^2 = 3$

Curves cut orthogonally , then $\frac{-2}{a} \times 3 = -1$

$a = 6$

18) Let $f(x) = 2x + 3 \times \frac{6}{x} = \frac{2x^2+18}{x}$

$f'(x) = \frac{x \times 4x - (2x^2+18) \times 1}{x^2} = \frac{2x^2-18}{x^2}$

$f"(x) = \frac{x^2 \times 4x - (2x^2-18) \times 2x}{x^4} = \frac{36x}{x^4} = \frac{36}{x^3}$

$f'(x) = 0 \Longrightarrow \frac{2x^2-18}{x^2} = 0 \Longrightarrow x = \pm 3$

$f(3)>0$ and $f(-3) < 0$

So $f(x)$is minimum when $x = 3$

Minimum value is = $2 \times 3 + 3 \times \frac{6}{3} = 12$

19) $f(x) = (x-1)^{\frac{1}{3}}(x-2)$

$f'(x) = (x-1)^{\frac{1}{3}}(1) + (x-2) \times \frac{1}{3}(x-1)^{\frac{-2}{3}} = (x-1)^{\frac{-2}{3}}\left(x-1+\frac{x-2}{3}\right)$

$= \frac{(x-1)^{\frac{-2}{3}}}{3}(3x-3+x-2)$

$= \frac{4x-5}{3(x-1)^{\frac{2}{3}}}$

$f'(x) = 0 \Longrightarrow x = \frac{5}{4}$

$f(1) = 0,\ \ f(9) = (9-1)^{\frac{1}{3}} \times 7 = 2 \times 7 = 14, \quad f\left(\frac{5}{4}\right) < 0$

Maximum value of $f(x)$ is 14

20) $y = be^{\frac{-x}{a}}$ crosses Y- axis $x = 0$, then $y = be^0 = b$

$\frac{dy}{dx} = b \times \frac{-1}{a}e^{\frac{-x}{a}}$

Slope of tangent = $\frac{dy}{dx}\ at\ (0,b) = \frac{-b}{a}e^0 = \frac{-b}{a}$

Equation of tangent is $y - b = \frac{-b}{a}(x-0)$

$ay - ab = -bx$

$bx + ay = ab \quad \frac{x}{a} + \frac{y}{b} = 1$

CASE STUDY QUESTIONS

I. Scientist want to know the Oil- Reserves in sea so they travel over the sea along the curve $f(x) = (x+1)^3 (x-3)^3$ by an aeroplane . A student of class XII discuss the characteristic of the curve.

Answer the questions on the basis of the information given above

(i) The first order derivative of the given function is

(a) $3(x+1)^2(x-3)^2$ (b) $6(x+1)^2(x-3)^2 (x-1)$

(c) $2(x-1)$ (d) None of these

(ii) The critical point of the given function are

(a) -1,1,3 (b) 1,3,-2

(c) 1,2 (d) None of these

(iii) The interval in which the given function is strictly increasing is

(a) $(1,3) \cup (3,\infty)$ (b) $(-\infty, -1) \cup (-1,1)$

(c) $(1,3) \cup (-1,\infty)$ (d) None of these

(iv) The interval in which the given function is decreasing is

(a) $(1,3) \cup (3,\infty)$ (b) $(-\infty, -1) \cup (-1,1)$

(c) $(1,3) \cup (-1,\infty)$ (d) None of these

II. Read the passage given below and answer the following questions.

To become fit and fine every person do some physical work or exercises. One morning two friends Mohan and Ahmed went for a morning walk in a park. They have decided to choose two different parabolic paths whose equations are $y^2 = 4x$(Mohan) and $x^2 = 4y$(Ahmed) respectively for their walk.

(i) The point at which both the paths meet is

(A) (0, 4)
(B) (4, 0)
(C) (0. 0) and (4, 4)
(D) they never meet

(ii) At the point (0, 0) the tangent to the curve $y^2 = 4x$ is

(A) parallel to the x-axis.
(B) parallel to the y-axis.
(C) parallel to the line y = 0.
(D) perpendicular to the line x = 0.

(iii) At the point (0, 0) the tangent to the curve $x^2 = 4y$ is

(A) parallel to the x-axis
(B) parallel to the y-axis
(C) perpendicular to the line y = 0.
(D) Parallel to the line x = 0

(iv) At the point (0, 0) the angle between the tangents to the curve $x^2 = 4y$ and $y^2 = 4x$ is

(A) 0
(B) $\pi/4$
(C) $\pi/2$
(D) $\pi/3$

(v) At the point (4, 4) the slope of the tangent to the curve $x^2 = 4y$ is

(A) 0
(B) 2
(C) -2
(D) 1/2

III. Read the passage given below and answer the following questions.

Let $P(x) = 4x^3 - 6x^2 - 72x + 30$ is the total profit function of a company, where x is the production of the company.

(i) What is the value of P'(x)?

(A) $12x^2 - 12x + 72$

(B) $12x^2 + 12x - 72$

(C) $x^2 - x - 6$

(D) $12x^2 - 12x - 72$

(ii) Determine the critical points of the profit function?

(A)-2, -3

(B) -2, 3

(C) 2,-3

(D)2,3

(iii) Check in which interval the profit is strictly increasing .

(A) $(-\infty, -2] \cup (3, \infty)$

(B) $(-\infty, -2] \cap [3, \infty)$

(C) $(-\infty, -2) \cup (3, \infty]$

(D) $(-\infty, -2) \cup (3, \infty)$

(iv) Check in which interval the profit is strictly decreasing .

(A) [-2, 3}

(B) [-2, 3)

(C) (– 2, 3)

(D) (-2, 3]

(v) What is the value of P''(x)?

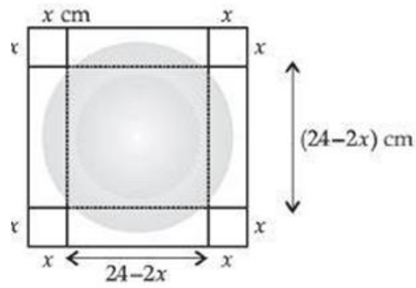
(A) 24x - 12

(B) 24x + 12

(C) -24x + 12
(D) -24x – 12

IV. An open box is to be made out of a piece of cardboard measuring 24 cm x 24 cm by cutting of equal squares from the corners and turning up the sides.

Based on this information answer all the following Questions.



(i) The volume V(x) of the open box is

a) $4x^3 - 96x^2 + 576x$
b) $4x^3 + 96x^2 + 576x$
c) $2x^3 - 48x^2 + 288x$
d) $2x^3 + 48x^2 + 288x$

(ii) The value of dV/dx is

a) $12(x^2+16x-48)$
b) $12(x^2-16x+48)$
c) $12(x^2-16x-48)$
d) $12(x^2+16x+48)$

(iii) The value of d^2V/dx^2 is

a) 24(x+8)
b) 12(x-4)
c) 24(x-8)
d) 12(x+4)

(iv) For what value of the height, the volume of the open box is maximum

a) 3 cm
b) 9 cm
c) 1 cm
d) 4cm

(v) The volume is minimum if

a) $dV/dx=0$ and $d^2V/dx^2=0$
b) $dV/dx=0$ and $d^2V/dx^2<0$
c) $dV/dx=0$ and $d^2V/dx^2>0$
d) None of these

V. Now a days Chinese and Indian troops are engaged in Aggressive melee, face-off skirmishes at locations near the disputed Pangong lake in Ladakh. One day a Helicopter of Enemy is Flying along the Curve represented by $y=\sqrt{5x-3}$ - 2. Indian soldiers are posted at a point P (2,3) . and try to hit it . But unfortunately bullet fire goes in straight line direction by just touching the helicopter . Based on the information answer the following questions.

i) The point on which tangents to the curve is perpendicular to the X axis.

(a) (0, 3/5)
(b) (3/5,0)
(c) (5/3, 0)
(d) None of these

ii) The equation of the tangents to the curve which are parallel to the y Axis

(a) 3x+ 5 =0
(b) 3x- 5 =0
(c) 5x-3 =0
(d) None of these

iii) The equation of the path travelled by the bullet Fire.

(a) $5x-2\sqrt{7}\,y = 15 - 4\sqrt{7}$.
(b) $2\sqrt{7}\,x -5y = 15 - 4\sqrt{7}$
(c) $2x – 5y = 15$
(d) None of these

iv) The slope of the Normal to the curve at x= 3/2 .

(a) 5/2
(b) -5/2
(c) -2/5
(d) None of these

ANSWERS

1	i) b (ii) a (iii) a (iv) b
2	(i) C (ii) B (iii) A (iv) C (v) B
3	(i) D (ii) B (iii) D (iv) C (v) A

4	(i) A (ii) B (iii) C (iv) D (v) C
5	(i) b (ii) c (iii) a (iv) c

ASSERTION & REASONING QUESTIONS

1. Assertion (A): The function y=$[x(x-2)]^2$ is increasing in (0,1)∪ $(2,\infty)$
   Reason(R):$\frac{dy}{dx}$ =0 when x=0,1,2

   A. Both A and R are true and R is the correct explanation of A
   B. Both A and R are true and R is not the correct explanation of A
   C. A is true but R is false
   D. A is false and R is true

2. Assertion (A): The equation of tangent to the curve y=sinx at the point (0,0) is y=x
   Reason (R): If y=sinx, then dy/dx at x=0 is 1

   A. Both A and R are true and R is the correct explanation of A
   B. Both A and R are true but R is NOT the correct explanation of A
   C. A is true but R is false
   D. A is false and R is true

SOLUTION

1. y=$[x(x-2)]^2$

   =$(x^2 -2x)^2$

   $\frac{dy}{dx}$=2(x^2-2x) (2x-2)

   $\frac{dy}{dx}$=0

   X=0,1,2

   Intervals are (-∞, 0), (0,1), (1,2), (2, ∞)

   f(x) is increasing in(0,1) U (2,∞) as f^I(x)>0 in (0,1) and (2,∞)

   Both A and R are true and R is not the correct explanation of A

   B is correct option

2. Y=sinx
   dy/dx= cosx
   Slope of tangent at (0,0) =cos0=1

   R is true

Equation of tangent at (0,0) is

$y-0=1(x-0)$

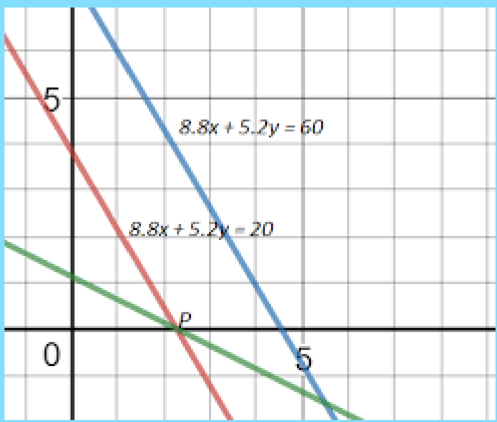
$y=x$

A is correct option

****

UNIT-IV

LINEAR PROGRAMMING



PART-7

LINEAR PROGRAMMING

What is Linear Programming?

A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum value) of a linear function (called objective function) of several variables (say x and y called decision variables) subject to the constraints that the variables are non negative and satisfy a set of linear inequalities (called linear constraints)

Mathematical Form of LPP

The general mathematical form of a linear programming problem may be written as, *Maximise or Minimise* $Z=c_1x+c_2y$ *subject to constraints are* $a_1x+b_1y\leq d_1$,$a_2x+b_2y\leq d_2$ *etc. and non-negative restrictions are* $x\geq 0$,$y\geq 0$

Some terms related to LPP

- **Constraints:** The linear inequalities or inequalities or restrictions on the variables of a linear programming problem are called constraints. The conditions $x\geq 0, y\geq 0$ are called non –negative restrictions

- **Optimisation Problem:** A Problem which seeks to maximise or minimise a linear function subject to certain constraints determined by a set of linear inequalities is called an optimisation problem . Linear programming problems are special types of optimization problems.

- **Objective Function:** A linear function of two or more variables which has to be maximised or minimised under the given restrictions in the form of linear inequations or linear constraints is called the objective function. The variables used in the objective function are called decision variables

- **Optimal Values:** The maximum or minimum value of an objective function is known as its optimal value

- **Feasible and infeasible regions:** The common region determined by all the constraints including non-negative constraints x, $y\geq 0$ of a linear programming problem is called the feasible region or solution region. Each point in this region represents a feasible choice. The region other than feasible is called an infeasible region.

- **Bounded and unbounded regions:** A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within a region otherwise it is said to be unbounded region
- **Optimal Solution:** A feasible solution at which the objective function has optimal value is called the optimal solution of the linear programming problem
- **Optimisation Technique:** The process of obtaining the optimal solution is called optimisation technique**.**

IMPORTANT THEOREMS.

- **Theorem I:** Let R be the feasible region (convex polygon) for a linear programming problem and Z= ax+by be the objective function When Z has an optimal value (maximum or minimum),where the variables x and y are subject to constraints described by linear inequalities , this optimal value must occur at a corner point (vertex) of the feasible region(A corner point of a feasible region is point of intersection of two boundary lines in the region)

- **Theorem 2: Let** R be the feasible region for a linear programming problem and Z=ax+by be the objective function. If R is bounded then the objective function Z has both a maximum and minimum value on R and each of these occurs at a corner point(vertex) of R

GRAPHICAL METHOD OF SOLVING LPP

The following steps are given below,

Step I: Find the feasible region of the linear programing problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point

Step II: Evaluate the objective function Z = ax +by at each corner point. Let M and m respectively denote the largest and smallest values of these points

Step III: When the feasible region is bounded, M and m are the maximum and minimum values of Z

MCQ QUESTIONS-LINEAR PROGRAMMING

1) Feasible region in the set of points which satisfy ___________

a) Objective function b) Some of the given constraints

c) All of the given constraints d) None

2) The first step in formulating a linear programming problem is ___________

a) Identify any upper or lower bound on the decision variables

b) State the constant as linear combination of the decision variables.

c) Understand any problem

d) Identify the decision variables.

3) Corner points of the feasible region for a Linear Programming Problem are (0,2), (3,0), (6,0), (6,8) and (0,5) and z=4x+6y be the objective function then maximum z – minimum z

a) 60 b) 48 c) 42 d) 18

4) Which of the following types of problems cannot be solved by Linear Programming Methods?

a) Diet problem b) Transportation problem c) Manufacturing problem d) Traffic Signal Control

5) Corner points of the feasible region determined by the system of linear constraints are (0,3), (1,1) and (3,0). Let z= p x + q y (p, q > 0). Find the condition on p and q so that the minimum of z occurs at (3,0) and (1,1) is –

a) p=2q b) p=q c) p=3q d) p=q/2

6) The solution of set of constraints $x+2y\geq 11, 3x+4y\leq 30, 2x+5y\leq 30, x, y\geq 0$ includes the point

a) (2,3) b) (3,2) c) (3,4) d) (4,3)

7) The graph of $x \leq 2$ and $y \geq 2$ is in ____________

a) Quadrants (i) and (ii)

b) Quadrants (ii) and (iii)

c) Quadrants (i) and (iii)

d) Quadrants (iii) and (iv)

8) The vertex of common graph of inequality is $2x+y \geq 2$ and $x-y \leq 3$ is

a) (0,0) b) (5/3, - 4/3) c) (5/3, 4/3) d) (-4/3, 5/3)

9) Find the maximum value of $z=4x+y$ subject to the constraints $x+y \leq 50, 3x+y \leq 90$, $x \geq 0$, $y \geq 0$ is

a) 120 b) 110 c) 50 d) None of these

10) Minimum value of $z= -3x+4y$ subject to $x+2y \leq 8$, $3x+2y \leq 12$, $x \geq 0$, $y \geq 0$ occurs at

a) (0, 0)

b) (4, 0)

c) {2, 3)

d) (0, 4)

11) One kind of cake requires 200g of flour and 25 g of fat and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

a) 25 b) 30 c) 20 d) None of these

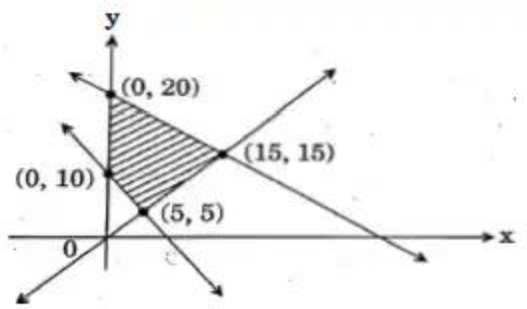
12) If a young man rides his motorcycle at 25 km/hour, he has to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/hour, the petrol cost increases at Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour.

a) 20km b) 30km c) 25km d) None of these

13) In the feasible region for a linear programming problem is ____________, then the optimal value of the objective function z= ax + by may or may not exist.

a) Bounded b) Unbounded c) In circled form d) In squared form

14) The feasible region of an LPP is shown in the figure. If z=3x+9y, then the minimum value of z occurs at?



a) (5,5)

b) (0, 10)

c) (0, 20)

d) (15, 15)

15) The linear programming problem minimize z=3x+2y subject to constraints x+y≥8, 3x+5y≤15, x≥0, y≥0 has

a) One solution b) no feasible solution c) two solutions d) infinitely many solutions

16) In an LPP, if the objective function z= ax + by has the same maximum value on two corner points of the feasible region then the number of points at which maximum value of z occurs is _____________

a) 0 b) 2 c) finite d) infinite

17) The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is Z = 4x + 3y. Compare the quantity in Column A and Column B

Column A	Column B
Maximum of Z	325

a) The quantity in column A is greater

b) The quantity in column B is greater

c) The two quantities are equal

d) The relationship cannot be determined on the basis of the information supplied

18) In a LPP, the objective function is always

(a) cubic

(b) Quadratic

(c) Linear

(d) constant

19) The linear inequalities or equations or restrictions on the variables of a linear programming problem are called ____ the conditions $x \geq 0$, $y \geq 0$ are called ____

(a) Objective functions, optimal value

(b) Constraints, non-negative restrictions

(c) Objective functions, non-negative restrictions

(d) Constraints, negative restrictions

20) A toy company manufactures two types of toys A and B. Demand for toy B is at most half of that if type A. Write the corresponding constraint if x toys of type A and y toys of type B are manufactured.

(a) $x/2 \leq y$

(b) $2y - x \geq 0$

(c) $x - 2y \geq 0$

(d) $x < 2y$

ANSWERS

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
c	**d**	**a**	**d**	**d**	**c**	**a**	**b**	**a**	**b**
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
b	**b**	**b**	**a**	**b**	**d**	**b**	**c**	**b**	**c**

CASE STUDY QUESTIONS

1. Corner points of the feasible region for an LPP are (0,0), (7,0), (0,5). Let Z=3x+4y be an objective function.

Based on the above information answer the following questions:

i).The minimum value of Z occurs at

a) (7,0)

b) (6,2)

c) (0,5)

d) (0,0)

ii)The maximum value of Z occurs at

a) (7,0)

b) (6,2)

c) (0,5)

d) (0,0)

iii).Maximum of Z- Minimum of Z is equal to

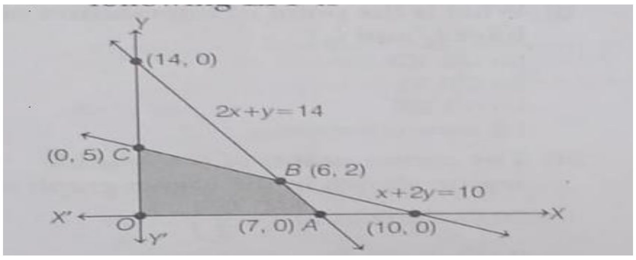
a) 26

b) 28

c) 21

d) 20

iv).If the objective function be Z= x+y, the maximum value of the following LPP is



a) 0

b) 7

c)8

d) 5

v).The feasible solution of LPP belongs to

a) First and second quadrant

b) First and third quadrant

c) Only second quadrant

d) Only first quadrant

2. An aeroplane can carry a maximum of 200 passengers. A profit of ₹.1000 is made on each executive class ticket and a profit of ₹.600 is made on each economy class ticket. The airline reserves at least 20 seats for the executive class, However atleast four times as many passengers prefer to travel by economy class, than by executive class. It is given that the number of executive class tickets is x and that of economy class ticket is y.

i) The maximum value of x+y is

a) 100

b) 200

c) 20

D) 80

ii) The relation between x and y is

a) $x<y$

b) $y>80$

c) $x\geq4y$

d) $y\geq4x$

iii) Which among these is not a constraint for this LPP?

a)$x\geq0$

b)$x+y\leq200$

c)$x\geq80$

d) $4x-y\leq0$

iv) The profit when x = 20 and y=80

a) ₹ 60000

b) ₹68000

c) ₹64000

d) ₹1,36,000

v) The maximum profit is ----------------

a) ₹1,36,000

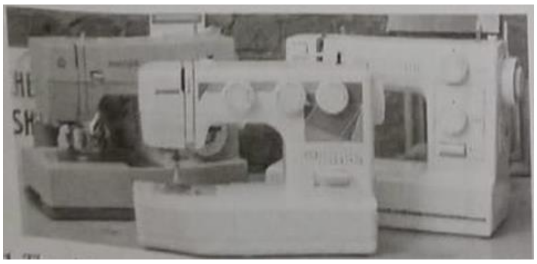
b) ₹1,28,000

c) ₹68000

d) ₹1,20.000

3. A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹5,760 to invest and has space for atmost 20 items for storage. An electronic sewing machine cost him ₹ 360 and a manually operated sewing machine ₹ 240. He can sell an electronic sewing

machine at a profit of ₹ 22 and a manually operated machine at a profit of ₹ 18. Assume that the electronic sewing machine he can sell is x and that of manually operated machine is y.



i. The objective function is ……………………….

a. Maximize Z = 360 x + 240 y

b. Maximize Z = 22x + 18 y

c. Minimize Z = 360 x + 240 y

d. Minimize Z = 22x + 18y

ii. The Maximum value of x + y is

a. 5760

b. 18

c. 22

d. 20

iii. Which of the following is not a constraint

a. $x+ y \geq 20$

b. $360 x + 240 y \leq 5760$

c. $x \geq 0$

d. $y \geq 0$

iv. The profit is maximum when (x,y) =

a. (5,15)

b. (8,12)

c. (12,8)

d. (15,5)

v) The maximum profit is ₹ …………………..

a) 5760

b) 392

c) 362

d) 290

ANSWERS
1. i). d ii). b iii). a iv). c v). d
2. i). b ii). d iii). c iv). b v). a
3. i). b ii). d iii). a iv). b v). b

LINEAR PROGRAMMING (ASSERTION REASON QUESTIONS)

DIRECTIONS :- (Q : NO 1 & 2) Each of these questions contains two statements : Assertion (A) and Reason (R) . Each of these questions also has four alternative choices , any one of which is the correct answer . You have to select one of the choices (a), (b) , (c) and (d) given below

(a) A is true ,R is true :R is a correct explanation for A.

(b) A is true ,R is true; R is not a correct explanation for A.

(c) A is true :R is false .

(d) A is false : R is true .

Q.1) Assertion (A) Objective function Z=13x -15y, is minimized subject to constraints $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$, $y \geq 0$ occur at corner point (0, 2) .

Reason (R) If the feasible region of the given LPP is bounded, then the maximum or minimum values of an objective function occur at corner points.

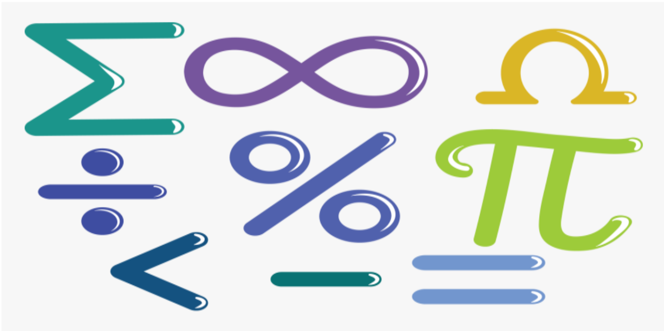
Q.2) Assertion (A) Maximize Z= 3x+ 4y subject to constraints: $x+ y \leq 1$, $x \geq 0$, $y \geq 0$. Then maximum value of Z is 4 .

Reason (R) If the shaded region is not bounded then maximum value cannot be determined .

Answers : Question .1 - option (a)

Question .2 – option (c) .

SAMPLE QUESTION PAPERS FOR REVISION



PRACTICE MAKES YOU PERFECT

KENDRIYA VIDYALAYA SANGATHAN

ERNAKULAM REGION

	SAMPLE QUESTION PAPER-1 **CLASS: XII** **Session: 2021-22** **Mathematics (Code-041) Term – 1** **Time Allowed: 90 minutes Maximum Marks: 40** **General Instructions:** **1. This question paper contains three sections – A, B and C. Each part is compulsory.** **2. Section - A has 20 MCQs, attempt any 16 out of 20.** **3. Section - B has 20 MCQs, attempt any 16 out of 20** **4. Section - C has 10 MCQs, attempt any 8 out of 10.** **5. There is no negative marking.** **6. All questions carry equal marks.**	
	SECTION A **In this section, attempt any 16 questions out of the Questions 1 - 20.** **Each Question is of 1 mark weightage**	
1	$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to a) Π b) π/3 c) -π/3 d) 2π/3	1
2	The value of K for which the function f(x)= $\begin{cases} kx^2 \text{ if } x \le 2 \\ 3 \text{ if } x > 2 \end{cases}$ is continuous at x=2 a) 3 b) 4 c) $\frac{3}{4}$ d) 1	1
3	If [a_{ij}] is a square matrix of order 3 and a_{ij} = i+2j ,then a_{31} is a) 7 b) 5 c) 0 d) 1	1
4	Value of k , for which $\begin{bmatrix} k & 6 \\ 6 & k \end{bmatrix}$ is a singular matrix a) 6 b) -6 c) 0 d) ±6	1
5	The function f(x) = tan x – x (a) Always increases (b) always decreases (c) Never increases (d) sometimes increases & sometimes decreases	1
6	Given that A is a square matrix of order 3 and I adj AI is 64, find IAI	1

	a) 8 b) -8 c) ±8 d) 0	
7	Let R be the relation in the set N given by R ={ (a,b) : a= b-2 , b>6 }. Choose the element in R a) (2,4) b) (3,8) c) (6,8) d) (8,7)	1
8	If $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 3 \end{bmatrix}$ then a+b+c+d is a) 9 b) 8 c) 0 d) 4	1
10	$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x is equal to a) 0, ½ b) 1, ½ c) ½ d) 0	1
11	Let the relation R in the set A = {1,2,3,4,5} given by R = {(a, b) :I a –b I is even}, then[1], the equivalence class containing 1 is a) {1, 2, 4} b) {1, 2, 3} c) {1, 3, 5} d) { }	1
12	Second derivative of y = log x is a) 1/x b) 1 c) 0 d) $-1/x^2$	1
13	If X and Y are matrices of order 2xn, 2xp and n = p, then order of the matrix 7 X – 5 Y is a) px2 b) 2xn c) nx3 d) p x n	1
14	If $y = \sin^{-1}x$, then $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$ is equal to a) 1 b) 2 c) -2 d) 0	1
15	If A is an invertible matrix of order 2, then det A^{-1} is equal to a) det A b) 1 c) 1/det A d) 0	1
16	The slope of normal to the curve y = $2x^2$ + 3 sin x at x = 0 is a) 3 b) 1/3 c) -3 d) -1/3	1
17.	For the matrix A =$\begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix}$, (adj A)' is a) $\begin{bmatrix} 2 & -4 \\ 3 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & -4 \\ 3 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$	1
18.	If y = sin x^2, then $\frac{dy}{dx}$ is a) cos x^2 b) 2x cos x^2 c) 2 sinx cosx d) 2x $\sin x^2$	1
19.	Maximum value of Z = 4x + 6y, subject to 3x + 2y ≤ 12, x+y≥ 4, x,y ≥ 0 is a) 36 b) 24 c) 16 d) 42	1

20. For all real values of x, the least value of $\frac{1-x+x^2}{1+x+x^2}$ is — 1

a) 0　　b) 1　　c) 3　　d) 1/3

SECTION B

In this section, attempt any 16 questions out of the Questions 21 - 40.
Each Question is of 1 mark weightage

21 **Let f : R → R be defined by f (x) = 1/x , x ∈ R. Then f is**

(a) one-one　　**(b) onto**
(c) bijective　　**(d) f is not defined**

22 **If $x = t^2$, $y = t^3$, then d^2y / dx^2 is**

(a)3/2　　**(b)3/4t**
(c)3/2t　　**(d)3/4**

23 A printing company prints two types of magazines A and B. The company earns ` 10 and ` 15 on each magazine A and B respectively. These are processed on three machines I, II & III and total time in hours available per week on each machine is as follows:

Magzine → ↓ Machine	A(x)	B(y)	Time available
I	2	3	36
II	5	2	50
III	2	6	60

The number of constraints is
(a) 3　　(b) 4
(c) 5　　(d) 6

24 **The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. $\cos^{-1}x$ is**

(a) 2　　**(b) $-1/2\sqrt{1-X^2}$**
(c) 2/x　　**(d) $1 - x^2$**

25 **Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is**

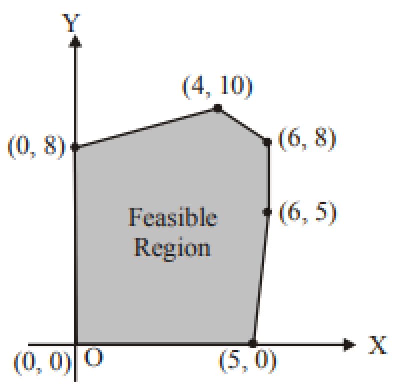
(a) 9　　**(b) 27**
(c) 81　　**(d) 512**

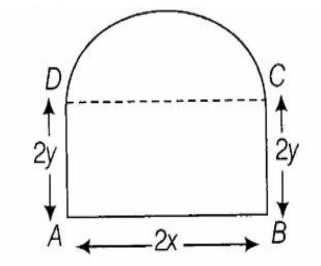
26 **The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is:**
(a) $[-1, \infty)$　　**(b) $[-2, -1]$**
(c) $(-\infty, -2]$　　**(d) $[-1, 1]$**

27 **The value of cot $[\cos^{-1}(7/25)]$ is**

(a) 25 /24　　**(b)25/7**

	(c)24/25 **(d) 7/24**	
28	**If A and B are invertible matrices, then which of the following is not correct?** **(a) adj A = |A|. A^{-1}** **(b) det$(A)^{-1}$ = [det $(A)]^{-1}$** **(c) $(AB)^{-1} = B^{-1}A^{-1}$** **(d) $(A + B)^{-1} = B^{-1} + A^{-1}$**	
29	**y = x $(x – 3)^2$ decreases for the values of x given by :** **(a) 1 < x < 3** **(b) x < 0** **(c) x > 0** **(d) 0 < x <3/2**	
30	**Let A = {1, 2, 3} and consider the relation** **R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)}.** **Then R is :** **(a) reflexive but not symmetric** **(b) reflexive but not transitive** **(c) symmetric and transitive** **(d) neither symmetric, nor transitive**	
31	**The function f (x) = cot x is discontinuous on the set** **(a) {x = nπ : n ∈ : Z}** **(b) {x = 2nπ : n ∈ Z}** **(c) {x = (2n + 1)π2: n ∈ Z}** **(d) {x = nπ2; n ∈ Z}**	
32	**Find the value of x, if** $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$ **= 0** **(a) -3/2** **(b) 3/2** **(c) 2** **(d) -2**	
33	**Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).** **Let F = 4x + 6y be the objective function.** **The Minimum value of F occurs at** **(a) (0, 2) only** **(b) (3, 0) only** **(c) the mid point of the line segment joining the points (0, 2) and (3, 0) only** **(d) any point on the line segment joining the points (0, 2) and (3, 0).**	
34	**Maximum slope of the curve y = $-x^3 + 3x^2 + 9x - 27$ is:** **(a) 0** **(b) 12** **(c) 16** **(d) 32**	
35	**If A is a square matrix such that A^2= I, then $(A–I)^3 + (A + I)^3–7A$ is equal to** **(a) A** **(b) I – A** **(c) I + A** **(d) 3A**	
36	**The value of the expression 2 $\sec^{-1} 2 + \sin^{-1}(1/2)$ is** **(a)π/6** **(b)5π/6** **(c)7π/6** **(d) 1**	

37	**Which of the following functions from Z into Z are bijections?** **(a) f (x) = x^3 (b) f (x) = x + 2** **(c) f (x) = 2x + 1 (d) f (x) = x^2 + 1**	
38	**If A and B are matrices of same order, then (AB′–BA′) is a** **(a) skew symmetric matrix (b) null matrix** **(c) symmetric matrix (d) unit matrix**	
39	**The points at which the tangents to the curve y = x^3 – 12x + 18 are parallel to x-axis are:** **(a) (2, –2), (–2, –34) (b) (2, 34), (–2, 0)** **(c) (0, 34), (–2, 0) (d) (2, 2), (–2, 34)**	
40	**If A is matrix of order m × n and B is a matrix such that AB′ and B′A are both defined, then order of matrix B is** **(a) m × m (b) n × n** **(c) n × m (d) m × n**	
	SECTION C **In this section, attempt any 8 questions. Each question is of 1-mark weightage.** **Questions 46-50 are based on a Case-Study.**	
41	**Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0) . Let Z = p x + q y, where p, q > 0. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is** **(a) p = 2 q (b) p = q/2 (c) p = 3 q (d) p = q**	1
42	**The equation of the normal to the curve $3x^2 - y^2$ = 8 which is parallel to the line x + 3y = 8 is** **(a) 3x – y = 8 (b) 3x + y + 8 = 0 (c) x + 3y ± 8 = 0 (d) x + 3y = 0**	1
43	**Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a.** **(a)$\frac{2a}{\sqrt{3}}$ (b) 2a$\sqrt{3}$ (c) a/3 (d) a/5**	1
44	**The feasible region for an LPP is shown shaded in the figure.**  **Let Z = 3 x – 4 y be the objective function. Minimum of Z occurs at** **(a) (0, 0) (b) (0, 8) (c) (5, 0) (d) (4, 10)**	1

45	The determinant $\begin{vmatrix} x & sin\theta & cos\theta \\ -sin\theta & -x & 1 \\ cos\theta & 1 & x \end{vmatrix}$ is (a) Independent of θ only (b) Independent of x only (c) Independent of both θ and x only (d) None of these	1
	CASE STUDY **An architect designs a building for a small company . The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi circular opening. The window is having a perimeter of 10 meter as shown in the figure.**  **Based on the above information answer the following:**	1
46	**If 2x and 2y represents the length and breadth of the rectangular portion of the window, then the relation between the variables is** **(a) 4y – 2x = 10- π (b) 4y = 10- (2-π)x** **(c) 4y = 10- (2+π)x (d) 4y – 2x = 10 + π**	1
47	**The combined area (A) of the rectangular region and semi circular region of the window expressed as a function of x is** **(a) A= 10x + (2+$\frac{\pi}{2}$) x^2 (b) A= 10x - (2+$\frac{\pi}{2}$) x^2** **(c) A= 10x - (2- $\frac{\pi}{2}$) x^2 (d) A = 4xy + $\frac{1}{2}\pi x^2$, where y= $\frac{5}{2}$ + $\frac{1}{4}$ (2+π)x**	1
48	**The maximum value of area of the whole window, A is** **(a) A = $\frac{50}{4+\pi}$ cm^2 (b) A = $\frac{50}{4+\pi}$ m^2 (c) A = $\frac{100}{4+\pi}$ m^2 (d) A = $\frac{50}{4-\pi}$ m^2**	1
49	**The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible. For this to happen, the length of the rectangular portion of the window should be** **(a) $\frac{20}{4+\pi}$ m (b) $\frac{10}{4+\pi}$ m (c) $\frac{4}{10+\pi}$ m (d) $\frac{100}{4+\pi}$ m**	1

50	**In order to get the maximum light input through the whole window, the area (in sq.m) of the only semi - circular opening of the window is** **(a)** $\frac{100\pi}{(4+\pi)^2}$ **(b)** $\frac{50\pi}{4+\pi}$ **(c)** $\frac{50\pi}{(4+\pi)^2}$ **(d) same as the area of rectangular portion of the window**	1

MARKING SCHEME

Q.NO	CORRECT OPTION	Q.NO	CORRECT OPTION	Q.NO	CORRECT OPTION	Q.NO	CORRECT OPTION	Q.NO	CORRECT OPTION
1	C	11	C	21	D	31	A	41	B
2	C	12	D	22	B	32	A	42	C
3	B	13	B	23	C	33	D	43	A
4	D	14	D	24	A	34	B	44	B
5	A	15	C	25	D	35	A	45	A
6	C	16	D	26	B	36	B	46	C
7	C	17	A	27	D	37	B	47	B
8	C	18	B	28	D	38	A	48	B
9	A	19	A	29	A	39	D	49	A
10	D	20	D	30	A	40	D	50	C

SAMPLE QUESTION PAPER-II

CLASS XII

MATHEMATICS

1. This question paper contains three sections – A B & C

2. Each part is compulsory.

3. Section A has 20 MCQ's, attempt any 16 out of 20.

4. Section B has 20 MCQ's, attempt any 16 out of 20.

5. Section C has 10 MCQ's, attempt any 8 out of 10.

6. There is no negative marking.

7. All questions carry equal marks.

SECTION A

In this section, attempt any 16 questions out of Questions 1 – 20.
Each Question is of 1 mark weightage

1.	Sin $\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to	
	A) 1/2	B) 1/3
	C)-1	D)1
2.	If the function defined by f(x) =$\begin{cases} \frac{4sinx}{x}, x < 0 \\ a - 2x, x \geq 0 \end{cases}$ is continuous at x =0, then a is:	
	A)4	B)-4
	C)2	D)1
3.	If A =[aij] is a square matrix of order 2 such that aij=$\begin{cases} 1, when\ i \neq j \\ 0, when\ i = j \end{cases}$, then A^2 is:	
	A)$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$	B)$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$
	C)$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$	D)$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
4.	The number of possible matrices of order 3x3 with each entry 0 or 1 is:	
	A)216	B)512
	C)1024	D)None

5.	The values of x for which f(x)=x^2-6x+9 is increasing is:	
	A)$[-3, \infty]$	B) $[3,\infty)$
	C) $(-\infty, 3]$	D) $(-\infty, \infty)$

6.	Given that A is a matrix of order 3 and lAl=-4, then ladjA l is equal to:	
	A)-4	B)4
	C)-16	D)16

7.	Let the relation R in the set A = {x/xϵw,0≤ $x \leq 12$} is given by R ={(a,b):((a-b) is a multiple of 4}.Then the set of all elements related to 2 is :	
	A) {1,5,9}	B) {0,4,8}
	C){0,2,4}	D) {2,6,10}

8.	If A= $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular matrix, then the value of x is:	
	A)-3	B)3
	C)4	D)-4
9.	If x=a (θ +sinθ) and y=a(1- cosθ) the value of $\frac{dy}{dx}$ at θ=$\frac{\pi}{2}$	
	A)1	B)-1
	C)∞	D)-∞
10.	Sin ($\tan^{-1} x$),where lxl<1, is equal to :	
	A)$\frac{X}{\sqrt{1-x^2}}$	B)$\frac{1}{\sqrt{1-x^2}}$
	C)$\frac{1}{\sqrt{1+x^2}}$	D)$\frac{x}{\sqrt{1+x^2}}$
11.	A relation R in set A ={1,2,3}is defined as R ={(1,1),(1,2),(3,3),(2,2)}.Which of the following ordered pair in R shall be removed to make it an equivalence relation in A	

12.	A point on the curve y=2x^2-6x-4 at which tangent is parallel to x-axis is:	
	A) (-3/2,17/2)	B) (-3/2,0)
	C) (3/2,-17/2)	D) (-3/2,-17/2)

13.	If $e^x + e^y = e^{x+y}, then$ $\frac{dy}{dx}$ is:	
	A)e^{y-x}	B)e^{x+y}
	C)$-e^{y-x}$	D) $2e^{x-y}$
14.	$If A = \begin{bmatrix} p^2 & 0 & 0 \\ 0 & 3q & 0 \\ 0 & 0 & 9 \end{bmatrix}$ is a scalar matrix, then values of p and q are:	
	A) p=3, q=3	B) p=-3, q=3
	C) p=$\pm$3,q=3	D) P=3, Q=-3
15.	If $\sin^{-1} x = \frac{\pi}{4}$, then the value of $\cos^{-1}(-x)$ is:	
	A)$\frac{3\pi}{2}$	B)$\frac{3\pi}{4}$
	C)$\frac{\pi}{2}$	D) None
16.	The graph of x≤ 2 and y≥ 2 will be situated in the:	
	A) third and fourth quadrant	B) third and first quadrant
	C) third and second quadrant	D) first and second quadrant
17.	The slope of normal to the curve x=a cos$^3\theta$, $y = asin^3\, \theta\; at\; \theta$=$\frac{\pi}{4}$ is:	
	A)2	B)1
	C)-1	D)-2

18.	If a matrix A, B and C are invertible matrix of same order, then $(ABC)^{-1}$ is:	
	A) CBA	B) $C^T B^{-1} A^T$
	C) $C^{-1}B^{-1}A^{-1}$	D) None
19.	Equation of tangent line to the curve y=2xsinx at the point $(\frac{\pi}{2},\pi)$ is:	
	A) $y=2\pi+x$	B) y=2x
	C) y=-2x	D) $y=-2x+\pi$

20.	If $\lvert A\rvert=3$ and $A^{-1}=\begin{bmatrix} 3 & -1 \\ -5/3 & 2/3 \end{bmatrix}$, then adjA is:	
	A) $\begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$	B) $\begin{bmatrix} 1 & -3 \\ -5 & 2 \end{bmatrix}$
	C) $\begin{bmatrix} -3 & 1 \\ 5 & 2 \end{bmatrix}$	D) $\begin{bmatrix} -9 & 3 \\ 5 & 2 \end{bmatrix}$

SECTION – B

In this section, attempt any 16 questions out of the Questions 21 - 40.

Each Question is of 1 mark weightage.

21. Let R be the relation in the set { 1, 2, 3, 4} given by:

R = { (1,2), (2, 2), (1, 1), (4, 4), (1,3), (3, 3), (3,2)]. Then

(a) R is reflexive and symmetric but not transitive.	**(b) R is reflexive and transitive but not symmetric**
(c) R is symmetric and transitive but not reflexive	**(d) R is an equivalent relation**

22. If x sin y + y cos x = π, then the value of y ''(0) is

(a) π	**(b) – π**
(c) 1	**(d) 0**

23. The corner points of the feasible region determined by the following system of linear inequalities. $2x + y \le 10$, $x + 3y \le 15$, $x , y \ge 0$ are (0,0), (5, 0), (3, 4) and (0,5). Let z = px + qy, where p, q > 0. Condition on p and q, so that the maximum of Z occurs at both 3, 4) and (0,5) is

(a) p = q	**(b) p = 2q**
(c) P = 3q	**(d) q = 3p**

24. The differential coefficient of sin {cos(x^2)} w.r.t x is

(a) – 2xsin(x^2) cos{cos(x^2)}	**(b) 2xsin(x^2) cos(x^2)**
(c) 2xsin(x^2) cos(x^2)cosx	**(d) xsin(x^2) cos{sin(x^2)}**

25. Let A = $\begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} & \mathbf{3} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix}$ and 10B = $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of α is:

(a) – 2	**(b) – 1**
(c) 2	**(d) 5**

26. The function $f(x) = cot^{-1}x + x$ increases in the interval

(a) $(1, \infty)$	(b) $(-1, \infty)$
(c) $(-\infty, \infty)$	(d) $(0, \infty)$

27. If $tan^{-1}\left(\frac{a}{x}\right) + tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$, then the value of x is:

(a) $\sqrt{ab}$	(b) $\sqrt{2ab}$
(c) $2ab$	(d) ab

28. If A is a square matrix of order 3 and $|A| = 5$, then $|2A'|$ is:

(a) – 10	(b) 10
(c) – 40	(d)40

29. Which of the following function is decreasing in $\left(0, \frac{\pi}{2}\right)$?

(a) . sin 2x	(b). tan x
(c) cos x	(d). cos 3x

30. If $A = \{ x\epsilon Z: 0 \leq x \leq 12\}$ and R is a relation inA given by $R = \{ (a, b): |a - b|\ is\ a\ multiple\ of\ 4\}$ .Then the set of all elements related to 1 is

(a) {1, 4, 8}	(b) {1, 5, 9}
(c) {2, 4, 6}	(d) {1, 3, 9}

31. The function $f(x) = x + |x|$ is continuous for

(a) $x \in (-\infty, \infty)$	(b) $x \in (-\infty, \infty) - \{0\}$
(c) only $x > 0$	(d) No value of x.

32. If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the value of k, so that $A^2 = 8A + kI$ is

(a) 4	(b) 5
(c) 6	(d) – 7

33. The Linear Programming Problem Minimize Z = 3x + 2y subject to

x + y ≥ 8, 3x + 5y ≤ 15, x ≥0, y ≥0 has

(a) one solution	(b) no feasible solution
(c) two solutions	(d) infinitely many solutions.

34. If 12 is divided into two parts such that the product of the square of one part and the fourth power of the second power is maximum, then its parts are:

(a) 5 and 7	(b) 6 and 6
(c) 3 and 9	(d) 4 and 8

35. If A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ **, then A2 – 5A+ 6 I is equal to**

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	(b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$	(d) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

36. If $cos^{-1}x = y$, then

(a) – 1 < y < 1	(b) $\frac{-\pi}{2} < y < \frac{\pi}{2}$
(c) $0 \le y \le \pi$	(d) $-\pi \le y \le \pi$

37. Which of the following functions from Z into Z are bijective?

(a) $f(x) = x^3$	(b) $f(x) = x + 2$
(c) $f(x) = 2x + 1$	(d) $f(x) = x^2 + 1$

38. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ then $(AB)^{-1}$ is

(a) $\frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$	(b) $\frac{1}{11}\begin{bmatrix} 14 & -5 \\ -5 & 1 \end{bmatrix}$
(c) $\frac{1}{11}\begin{bmatrix} 1 & 5 \\ 5 & 14 \end{bmatrix}$	(d) $\frac{1}{11}\begin{bmatrix} 1 & -5 \\ -5 & 14 \end{bmatrix}$

39. The line y = x + 1 is a tangent to the curve $y^2 = 4x$, then the point of contact is

(a) (1, 2)	(b) (2, 1)
(c) (1, – 2)	(d) (– 1, 2)

40. . If $A = \begin{bmatrix} \alpha & b \\ b & a \end{bmatrix}$ and $B = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ then

(a) $\alpha = a^2 + b^2$, $\beta = ab$	(b) $\alpha = a^2 + b^2$, $\beta = 2ab$
(c) $\alpha = a^2 + b^2$, $\beta = a^2 - b^2$	(d) $\alpha = 2ab$, $\beta = a^2 + b^2$

SECTION C

In this section, attempt any 8 questions

Each question is of 1 mark weightage.

Questions 46-50 are based on a case study

41. **Corner points of the feasible region for an LPP are (0,2) ,(3,0),(6,0),(6,8) and (0,5) .Let F = 4x +6y be the objective function, then maximum of F – minimum of F =**

(a) 60	(b) 48
(c) 42	(d) 18

42. **The maximum value of $x^{\frac{1}{x}}$ x>0 is**

(a) $e^{\frac{1}{e}}$	(b) $\left(\frac{1}{e}\right)^{e}$
(c) 1	(d) None of these

43. **The point on the curve $y^2 = x$, where the tangent make an angle 45^0 with X axis is**

(a) $\left(\frac{1}{2},\frac{1}{4}\right)$	b) $\left(\frac{1}{4},\frac{1}{2}\right)$
(c) (4,2)	(d) (2,-2)

44. **A linear programming problem is as follows:**

Minimize$Z = 3x + 9y$, subject to constraints $x + 3y \le 60$, $x + y \ge 10$,

$x \ge 0$, $y \ge 0$. Then the minimum value of Z is

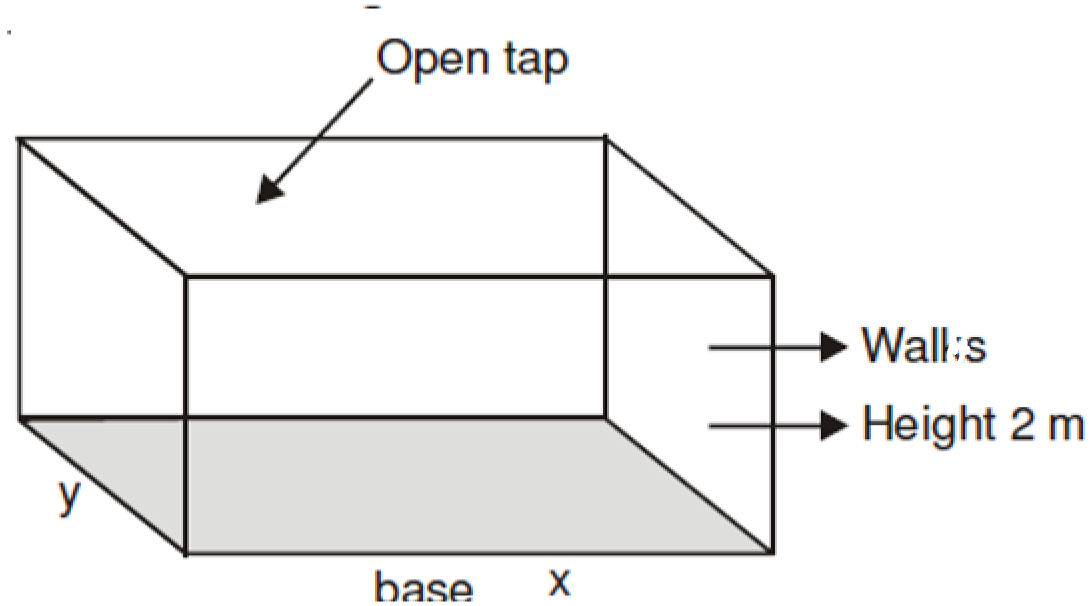
(a) 30	(b) 40
(c) 90	(d) 180

45. The maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+sin\theta & 1 \\ 1 & 1 & 1+sin\theta \end{vmatrix}$ is

a) $\frac{1}{2}$	((b) 0
(c) 1	(d) $\frac{1}{4}$

CASE STUDY

A building constructor, construct a water tank of capacity 8 cubic meter with rectangular base and rectangular side, with open top. The depth of the tank is 2m . The cost of construction of base is Rs 70 per square meter and cost of construction for side wall is Rs 45 per sq. meter.



On the basis of given information answer the following

46. If the length of the tank is x meter, breadth of tank is y meter then the relation between x and y is

(a) $y = \frac{4}{x}$	(b) $y = \frac{8}{x}$
(c) $y = \frac{1}{x}$	(d) $y = x^2$

47. Cost of building the four walls of the tank at the rate Rs 45 is

(a) 45x +90 y	(b) 90x +180 y
(c) 180 x +90 y	(d)) 180(x + y)

48. Total cost of building the tank is

(a) 70xy + 45 (x+y)	(b) 70 xy + 90 (x+y)
(c) 70xy + 40 (x+y)	(d) 70 xy + 180 (x +y)

49. Total cost of tank is minimum when the length of the tank is

(a) 3 m	(b) 2m
(c) 4m	(d) 6m

50. Minimum cost of the construction of the tank at the given rate is

(a) 1200	(b) 200
(c) 1000	(d) 700

ANSWERS

1(D)	2(A)	3(D)	4(B)	5(B)	6(D)	7(D)	8(B)	9(A)	10(D)
11(B)	12(C)	13(C)	14(C)	15(B)	16(D)	17(B)	18(C)	19(B)	20(A)
21(b)	22(a)	23(d)	24(a)	25(d)	26(c)	27(a)	28(d)	29(c)	30(b)
31(a)	30(d)	33(b)	34(d)	35(c)	36(c)	37(b)	38(a)	39(a)	40(b)
41(a)	42(a)	43(b)	44(d)	45(a)	46(a)	47(d)	48(d)	49(b)	50(c)